Due on Thursday March 23d–after Spring Break

Re-do Miterm 1 problems you missed or did not do from Part B

From Artin

Chapter 7 (p. 223): 5.1(c), 5.2, 5.3, 5.4, 5.6, 5.7.

Others:

1. Let $\alpha \in S_9$ denote the permutation $(1\ 9)(2\ 8)(3\ 7)(4\ 6)$. Is $\alpha$ even or odd.
2. For $1 \leq r \leq n$ compute the number of $r$-cycles in $S_n$.
3. Show that an $r$-cycle is even if and only if $r$ is odd.
4. Give an example of $\alpha, \beta, \gamma \in S_5$ with $\alpha \beta = \beta \alpha$, $\alpha \gamma = \gamma \alpha$ but $\beta \gamma \neq \gamma \beta$
5. If $\alpha \in S_n$ is an $r$-cycle is $\alpha^k$ an $r$-cycle?
6. (a) Show that $\alpha \in S_n$ has order 2 if and only if its cycle decomposition is a product of commuting transpositions.
   (b) Prove that the order of an element in $S_n$ equals the least common multiple of the lengths of the cycles in its cycle decomposition.
7. Let $p$ be a prime and let $H_p$ denote the set of matrices of $SL(3, \mathbb{Z}/p\mathbb{Z})$ given by:
   $$\left\{ \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} : x, y, z \in \mathbb{Z}/p\mathbb{Z} \right\}.$$  
   (a) Prove that $H_p$ is a subgroup of $SL(3, \mathbb{Z}/p\mathbb{Z})$ of order $p^3$ (The mod $p$ Heisenberg group).
   (b) Identify $H_2$.
   (c) Prove directly (i.e. without using the class equation) that $Z(H_p)$ is non-trivial. What is it?
   (d) Compute the abelianization of $H_p$; i.e. $H_p/[H_p, H_p]$ where $[H_p, H_p]$ is the commutator subgroup of $H_p$.
8. Let $S$ be the collection all functions $f : \mathbb{R} \to \mathbb{R}$ that have derivatives of all orders. Define a binary operation $+$ on $S$ by $(f + g)(x) = f(x) + g(x)$.
   (i) Prove that $(S, +)$ is a group.
   (ii) Define $\phi : (S, +) \to (\mathbb{R}, +)$ by $\phi(f) = f'(0)$. Prove that $\phi$ is a homomorphism. Is $\phi$ one-to-one?
   (b) Let $C = \{f : [0, 1] \to \mathbb{R} : f \text{ is continuous}\}$, i.e. the set of continuous real-valued functions with domain $[0, 1]$. As in (a) define a binary operation $+$ on $C$ by $(f + g)(x) = f(x) + g(x)$ which makes $(C, +)$ a group (you do not need to prove this).
   Define $\sigma : (C, +) \to (\mathbb{R}, +)$ by
   $$\sigma(x) = \int_0^1 f(x)dx.$$
(i) Prove that $\sigma$ is a homomorphism.
(ii) Give an example of a non-zero function in the kernel of $\sigma$. 