Homework 1 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Wednesday 8th October (Due to Labour Day)

(1) For each of the following statements either provide a proof or a counterexample
  (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
  (b) $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.

(2) Let R be a relation satisfying on X satisfying
  (a) xRx for every $x \in X$. (R is reflexive)
  (b) if xRy then yRx. (R is symmetric)
  (c) if xRy and yRz then xRz. (R is transitive)

  such a relation is called an *equivalence relation*. Let

$$[x] := \{y : yRx\}$$

  Prove that if $[x] \cap [y] \neq \varnothing$ then $[x] = [y]$.
  [Remark: If we replace symmetry by
  (b) if xRy and yRx then $x = y$
  then we obtain a *partial order*.]

(3) For each of the following statements either provide a proof or a counterexample
  (a) $f(A \cap B) = f(A) \cap f(B)$.
  (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
  (c) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
  (d) if $g \circ f$ is onto then f and g are both onto.
  (e) if $g \circ f$ is one-to-one then f and g are one-to-one.

(4) Show that a set is infinite if and only if it can be put into one-to-one correspondence with a proper subset of itself.
  [Hint: Show that $\mathbb{N}$ can be put into one-to-one correspondence with a subset of itself and then use this.]