

## HOMWORK 10 FOR M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE MONDAY NOVEMBER 22ND AT 9 AM

(1) Let

$$C^1([a, b]) = \{f : f \text{ is differentiable on } [a, b] \\ \text{and } f' \text{ is continuous}\}$$

Let

$$Lip([a, b]) = \{f : \text{there exists } L, \delta > 0 \text{ such that} \\ |f(x) - f(y)| < L|x - y| \text{ for all } |x - y| < \delta\}$$

Show that if  $f \in C^1([a, b])$  then  $f \in Lip([a, b])$ .

(2) Suppose  $f$  is integrable on  $[a, b]$  and  $g \in C^1([a, b])$  is non-decreasing. Prove that

$$\int_a^b f dg = \int_a^b fg'$$

(3) Suppose  $x(t)$  is a solution to the initial value problem

$$\frac{dx}{dt} = f(x, t) \quad x(t_0) = x_0$$

where  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  is continuous. Prove

$$x(t) = \int_{t_0}^t f(x(s), s) ds + x_0$$