Homework 7 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday November 1st at 9 am

(1) Let $\pi = (x_i)_{i=0}^n$ be a partition of $[-a, a]$. Show that there exists a partition $\pi' = (y_j)_{j=0}^{2m}$ with the properties
  Symmetry: for all $0 \leq j \leq 2m$ $y_j = -y_{2m-j}$.
  Refinement: $\pi \leq \pi'$.

(2) Suppose $f : [-a, a] \to \mathbb{R}$ is odd, i.e. $f(-x) = -f(x)$. Suppose that $\int_{-a}^{a} f$ exits. Show that $\int_{-a}^{a} f = 0$ using the definition of the Riemann integral.

(3) Suppose $f : [-a, a] \to \mathbb{R}$ is even, i.e. $f(-x) = f(x)$. Suppose that $\int_{-a}^{a} f$ exits. Show that $\int_{-a}^{a} f = 2\int_{0}^{a} f$ using the definition of the Riemann integral.

(4) Show that for $f : [a, b] \to \mathbb{R}$, $g : [a, b] \to \mathbb{R}$, and $\alpha, \beta \in \mathbb{R}$ we have

$$S(\alpha f + \beta g, \pi, \sigma) = \alpha S(f, \pi, \sigma) + \beta S(g, \pi, \sigma)$$

for all partitions π of $[a, b]$ and all samples associated to π. Hence show that if f and g are Riemann integrable on $[a, b]$ then $\alpha f + \beta g$ is Riemann integrable and

$$\int_a^b (\alpha f + \beta g) = \alpha \int_a^b f + \beta \int_a^b g$$