Homework 9 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday November 15th at 9 am

(1) Let $f, g : I \to \mathbb{R}$ for some interval $I \subset \mathbb{R}$. Prove that if f and g are differentiable at x then $f \cdot g$ is differentiable at x and

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

(2) Show that for $n \in \mathbb{N}$

$$y^n - x^n = (y - x)(y^{n-1} + y^{n-2}x + \cdots + yx^{n-2} + x^{n-1}).$$

Let $n \in \mathbb{Z}$ and define $f(x) = x^n$. Prove that f is differentiable on $\mathbb{R}$ (or $\mathbb{R}\backslash\{0\}$ if $n < 0$) and show that $f'(x) = nx^{n-1}$.

(3) Show that if $E : \mathbb{R} \to \mathbb{R}$ satisfies $E(x + y) = E(x) \cdot E(y)$ then E is differentiable at x if and only if it is differentiable at 0 and then $E'(x) = E(x) \cdot E'(0)$.