

MATH 343K EXAM 1

Name: _____
UT EID: _____

INSTRUCTIONS

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 20 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- All solutions must be in the form of complete sentences.

FOR INSTRUCTOR'S USE

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Total	_____

Question 2

1. Define what it means for $x \in S$ to be an identity element for the binary structure $\langle S, * \rangle$.
2. Prove that an identity element of a binary structure, if it exists, is unique.
3. Suppose $e \in S$ is an identity element for $\langle S, * \rangle$. Suppose $\varphi : S \rightarrow T$ is an isomorphism of binary structures $\langle S, * \rangle$ and $\langle T, \star \rangle$. Show that $\langle T, \star \rangle$ has an identity element.

Question 3

1. Let $\varphi : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_{10}$. Suppose $\varphi(2) = 5$. Can φ be an automorphism of $\langle \mathbb{Z}_{10}, +_{10} \rangle$? Justify your answer.
2. Determine the number of automorphisms of $\langle \mathbb{Z}_{10}, +_{10} \rangle$.
3. Let $\varphi : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_{10}$ be an automorphism of $\langle \mathbb{Z}_{10}, +_{10} \rangle$ with $\varphi(7) = 1$. Find $\varphi(1)$. Justify your answer.
4. Determine the number of automorphisms of $\langle \mathbb{Z}, + \rangle$.

Question 4

1. Define the order of an element $x \in G$ of a group $\langle G, * \rangle$.
2. Determine the order of the element $525 \in \mathbb{Z}_{3465}$. Hint: $3465 = 3^2 \times 5 \times 7 \times 11$ and $525 = 3 \times 5^2 \times 7$.
3. Find all the subgroups of \mathbb{Z}_{84} and draw the subgroup diagram. Hint: $84 = 2^2 \times 3 \times 7$