

# MATH 343K EXAM 2

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| Name: _____   |
| UT EID: _____ |

## INSTRUCTIONS

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 20 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- All solutions must be in the form of complete sentences.

## FOR INSTRUCTOR'S USE

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| Question 1 | _____ |
| Question 2 | _____ |
| Question 3 | _____ |
| Question 4 | _____ |
| Total      | _____ |

## Question 1

1. Let  $H_i$ , for  $i$  in an index set  $I$ , be subgroups of  $\langle G, * \rangle$ . Define

$$H = \bigcap_{i \in I} H_i.$$

Prove that  $H$  is a subgroup of  $G$ .

2. Find, up to isomorphism, all abelian groups of order 48.

3. Show that  $\mathbb{Z}_{12} \times \mathbb{Z}_3$  is not isomorphic to  $\mathbb{Z}_6 \times \mathbb{Z}_6$ .

## Question 2

1. Define what it means for a permutation  $\sigma \in S_n$  to be an even permutation.
  
  
  
  
  
  
  
  
  
  
2. Write the following permutation as a product of disjoint cycles and say whether it is an even or an odd permutation. *Justify your answer.*

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 1 & 2 & 4 & 7 & 6 \end{pmatrix}$$

3. Prove that a cycle  $\sigma$  of length  $n$  is
  - (a) an even permutation if  $n$  is odd.
  - (b) an odd permutation if  $n$  is even.

*You need only write  $\sigma$  in the required form.*

4. State Cayley's Theorem.

## Question 3

1. Let  $H$  be a subgroup of  $\langle G, * \rangle$ . Define  $aH$ , the left coset of  $H$  containing  $a$ .
2. Prove that if  $aH \cap bH \neq \emptyset$  then  $aH = bH$ . *Hint: By symmetry it suffices to show that  $aH \subseteq bH$ .*
3. Find the left cosets of  $H = \{0, 3\}$  in the group  $\langle \mathbb{Z}_6, +_6 \rangle$ .
4. State Lagrange's Theorem.
5. Prove that every group  $G$  of prime order is cyclic.

## Question 4

1. Define *Frieze group*.
2. Describe the different isometries that are allowed to appear in a Frieze group.
3. Give the *crystallographic restriction* and explain when it applies.
4. There is one type of Frieze group which is abelian but not cyclic. Describe its generators and draw a Frieze pattern which has this group as its largest group of symmetries.