

# MATH 343K FINAL EXAM

Name: _____
UT EID: _____

## INSTRUCTIONS

- Please put your name and UT EID in the space provided.
- There are 8 questions each worth 20 points.
- You have 180 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- All solutions must be in the form of complete sentences.

## FOR INSTRUCTOR'S USE

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Total	_____







## Question 4

1. Express the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$$

in the form of disjoint cycles.

2. Express the permutation  $(145)(78)(257)$  in the form

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

3. Define what it means for a permutation  $\sigma$  to be an *even* permutation.

4. Is the permutation

$$\sigma = (134)(2468)(27)(43251)(783)(1342)(18)$$

even or odd? Justify your answer.

Hint: It is not necessary to compute  $\sigma$ .

## Question 5

1. Define what it means for  $\phi : G \rightarrow G'$  to be a *homomorphism*.
2. Show that  $\phi : \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $\phi(m) = 5m$  is a homomorphism.
3. Let  $\phi : G \rightarrow G'$  be a homomorphism. Show that  $\phi(x^{-1}) = \phi(x)^{-1}$ .
4. Let  $\phi : G \rightarrow G'$  be a homomorphism. Show that  $\phi(e_G) = e_{G'}$ .

## Question 6

1. Define the index of a subgroup  $H$  in  $G$ ,  $(G : H)$ .

2. Find all of the left cosets of  $H = \langle 6 \rangle$  in  $\langle \mathbb{Z}_{30}, +_{30} \rangle$ .

3. Give the coset which corresponds to

$$(3 + H) + (8 + H).$$

4. Determine the order of  $8 + H$  in the quotient group  $G/H$ .

## Question 7

1. Let  $\phi : G \rightarrow G'$  be a homomorphism.

Define  $\ker(\phi)$  and show it is a subgroup.

2. Define what it means for  $H$  to be a *normal subgroup* of  $G$ .

3. Show that the kernel of  $\phi$  is a normal subgroup.

## Question 8

1. Describe the 10 element group of isometries of a pentagonal prism.
2. Give Burnside's Formula
3. Determine the number of colorings of a pentagonal prism using 7 colors if each color can only be used on one face.
4. Determine the number of colorings of a pentagonal prism using 7 colors if colors may be used on more than one face.