

## HOMWORK 1 FOR M343K

- Please label your homework clearly with your name.
- Homework must be neatly written and stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.
- All solutions must take the form of complete sentences.

DUE TUESDAY SEPTEMBER 13TH

(1) Prove that

$$(\cos(\theta_1) + i \sin(\theta_1)) \cdot (\cos(\theta_2) + i \sin(\theta_2)) = \cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$$

using the definition of complex multiplication and trig identities.

- (2) Let  $c, d \in \mathbb{R}^+$  be arbitrary. Show that binary structure  $\langle \mathbb{R}_c, +_c \rangle$  is isomorphic to the binary structure  $\langle \mathbb{R}_d, +_d \rangle$ .
- (3) Let  $U_6 = \{z : z^6 = 1\}$  be the set of 6th roots of unity. Show that the binary structure  $\langle U_6, \times \rangle$  is isomorphic to  $\langle \mathbb{Z}_6, +_6 \rangle$ .
- (4) Can there be an isomorphism  $\phi$  of  $U_6$  with  $\mathbb{Z}_6$  such that  $\phi(e^{i\pi}) = 2$ ? Explain.
- (5) How many different binary operations can be defined on an set  $S$  with  $n$  elements?

*Remark:* Some of these operations may give isomorphic binary structures. By different binary operations I mean different functions  $* : S \times S \rightarrow S$ .

- (6) How many different *commutative* binary operations can be defined on a set  $S$  with  $n$  elements?

*Remark:* Some of these operations may give isomorphic binary structures. By different binary operations I mean different functions  $* : S \times S \rightarrow S$ .