Homework 3 for M343K

- Please label your homework clearly with your name.
- Homework must be neatly written and stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.
- All solutions must take the form of complete sentences.

Due Thursday September 29th

(1) Show that for all $a, b \in \mathbb{Z}$ the subset
$$\{ma + nb : m, n \in \mathbb{Z}\}$$
is a subgroup of $< \mathbb{Z}, + >$.

(2) An *automorphism* of G is an isomorphism $\varphi : G \to G$. Every group has the identity as an automorphism.
   (a) Show that $\varphi(0) = 0$, $\varphi(1) = 2$ and $\varphi(2) = 1$ is an automorphism of $\mathbb{Z}_3$.
   (b) Show that $\varphi(0) = 0$, $\varphi(1) = 5$, $\varphi(2) = 4$, $\varphi(3) = 3$, $\varphi(4) = 2$, and $\varphi(5) = 1$ defines and automorphism of $\mathbb{Z}_6$.

(3) For each of the following groups determine the number of automorphisms.
   (a) $\mathbb{Z}_7$
   (b) $\mathbb{Z}_8$
   (c) $\mathbb{Z}_{36}$.

   Hint: Under an automorphism a generator must go to a generator and an automorphism is completely determined by its value on a generator. Indeed, if $\varphi : G \to H$ satisfies the homomorphism property and G is cyclic with generator a then φ is completely determined by $\varphi(a)$.

(4) For each of the following finite cyclic groups find the subgroup diagram
   (a) $\mathbb{Z}_{36}$
   (b) $\mathbb{Z}_{100}$
   (c) $\mathbb{Z}_{24}$

(5) Let $< G, * >$ be a group. Show that if $a * b \in G$ has order n then $b * a \in G$ also has order n.