Homework 5 for M343K

- Please label your homework clearly with your name.
- Homework must be neatly written and stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.
- All solutions must take the form of complete sentences.

Due Tuesday October 18th

(1) Complete the following table so that no row or column contains a repeated element but such that the operation is not associative.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a				
b	b				
c	c				
d	d				

Prove that the operation is not associative. *Remark*: We have seen that if you complete a 3×3 or 4×4 table so that there is an identity element and no row or column contains a repeated element then the operation is automatically associative.

(2) The *right regular representation* of $< G, * >$ as a subgroup of S_G is given by $\varphi : G \to S_G$, which is defined

$$\varphi(g) = \rho_{g^{-1}} \qquad \text{where} \qquad \rho_g(x) = x * g$$

Show that φ is injective and that it has the homomorphism property.

(3) Let $\sigma \in S_A$ be a permutation. Show that the relation $\sim_\sigma$ on A defined by

$$x \sim_\sigma y \quad \text{if and only if} \quad \exists m \in \mathbb{Z},\ x = \sigma^m(y)$$

is an equivalence relation.

(4) Prove that any finite number of disjoint cycles commute.

(5) Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 5 & 1 & 8 & 6 & 2 & 3 & 4 \end{pmatrix} \in S_8$$

Find the partition of $\{1, \ldots, 8\}$ in to orbits of σ and hence write σ as a product of disjoint cycles.