Homework 8 for M343K

- Please label your homework clearly with your name.
- Homework must be neatly written and stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.
- All solutions must take the form of complete sentences.

Due Thursday December 1st

(1) Show that every subgroup H of a commutative group G is normal.
(2) Let $G = \mathbb{Z}_{60}$ and let $H =< 12 >$. Find the order of $28 + H$ in G/H.
(3) Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$ and let $H = \langle (1,1) \rangle$. Find the partition of G into cosets and give the group table for the resulting four element group.
(4) Let G be an commutative group. Show that
$$\mathrm{Tor}(G) = \{g \in G : g \text{ has finite order } \}$$
is a subgroup of G. This is called the **torsion subgroup** of G.
(5) Show that if G is not commutative then $\mathrm{Tor}(G)$ defined above need not be a subgroup by taking G to be the group of plane isometries.
(6) Let G be an commutative group. Show that the factor group $G/\mathrm{Tor}(G)$ has no elements of finite order except the identity. Such a group is called **torsion free**.
(7) Let H be a finite index normal subgroup of G, i.e. a normal subgroup with $(G : H) = m$ for some $m \in \mathbb{N}$. Show that for every $a \in G$ we have $a^m \in H$. Hint: Consider $aH \in G/H$.