

HOMWORK 9 FOR M343K

- Please label your homework clearly with your name.
- Homework must be neatly written and stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.
- All solutions must take the form of complete sentences.

DUE THURSDAY DECEMBER 8TH

- (1) For a subgroup H of G we can define the normalizer of H in G , $N_G(H)$, by

$$N_G(H) := \{g \in G : gHg^{-1} = H\}.$$

Show that $N_G(H)$ is a subgroup and $H \subseteq N_G(H)$. Show that H is a normal subgroup of $N_G(H)$.

- (2) Show that the number of subgroups conjugate to H is $(G : N_G(H))$. Hint: G acts on the set of subgroups of G by conjugation $g \cdot H = gHg^{-1}$.
- (3) Suppose a cube is to be painted so that each face is a different color. If eight colors are available how many distinguishable ways of painting the cube are there ?
- (4) Suppose a cube is to be painted so that each face is a single color. If eight colors are available how many distinguishable ways of painting the cube are there ?