Homework 1 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday January 31st

(1) For each of the following statements either provide a proof or a counterexample
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
   (b) $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.

(2) Let R be an *equivalence relation* on X. Let

$$[x] := \{y : yRx\}$$

denote the *equivalence class* of $x \in X$. Prove that if $[x] \cap [y] \neq \varnothing$ then $[x] = [y]$.

(3) Let U be a set. Let $X = \mathcal{P}(U)$. Show that the relation $\preceq$ on X defined by

$$A \preceq B \text{ if and only if } A \subseteq B$$

is a *partial order* on X.

(4) Let $f : X \to Y$ and $g : Y \to Z$ be functions. For each of the following statements either provide a proof or a counterexample.
   (a) $f(A \cap B) = f(A) \cap f(B)$.
   (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
   (c) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
   (d) if $g \circ f$ is onto then f and g are both onto.
   (e) if $g \circ f$ is one-to-one then f and g are one-to-one.