

HOMWORK 2 FOR M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE MONDAY FEBRUARY 7TH AT 10 AM

- (1) Show that a countable union of countable sets is countable. I.e. If I is a countable index set and for each $i \in I$ A_i is a countable set then

$$\bigcup_{i \in I} A_i$$

is countable.

- (2) Let X and Y be finite sets.
(a) Show that

$$|\mathcal{P}(X)| = 2^{|X|}.$$

- (b) Let

$$Y^X = \{f : X \rightarrow Y\}.$$

Show that

$$|Y^X| = |Y|^{|X|}.$$

- (c) How many one to one functions are there from X to Y ?

- (3) Let $|\cdot|$ be the absolute value on \mathbb{R} . Prove the following properties

- (a) $|x| = 0$ if and only if $x = 0$.
- (b) $|x - y| = |y - x|$ for all $x, y \in \mathbb{R}$.
- (c) $|x - z| \leq |x - y| + |y - z|$ for all $x, y, z \in \mathbb{R}$.
- (d) $|x - y| \geq ||x| - |y||$ for all $x, y \in \mathbb{R}$.
- (e) $|xy| = |x||y|$

- (4) Give examples of each of the following:

- (a) Non-empty bounded intervals $I_n \subseteq \mathbb{R}$ with $I_{n+1} \subseteq I_n$ and $\bigcap_{n=1}^{\infty} I_n = \emptyset$.
- (b) Non-empty closed intervals $I_n \subseteq \mathbb{R}$ with $I_{n+1} \subseteq I_n$ and $\bigcap_{n=1}^{\infty} I_n = \emptyset$.
- (c) Non-empty, closed, and bounded intervals $I_n \subseteq \mathbb{Q}$ with $I_{n+1} \subseteq I_n$ and $\bigcap_{n=1}^{\infty} I_n = \emptyset$ (in \mathbb{Q}).