Homework 3 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday February 14th at 10 am

(1) Prove that if $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are convergent sequences then
$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

(2) Prove that if $(a_n)_{n=1}^{\infty}$ is a convergent sequence and $c \in \mathbb{R}$ is a constant then
$$\lim_{n\to\infty}(c \cdot a_n) = c \cdot \lim_{n\to\infty} a_n$$

(3) Suppose that $(a_n)_{n=1}^{\infty}$ is a convergent sequence and $(b_n)_{n=1}^{\infty}$ diverges to $+\infty$. Show that
$$\lim_{n\to\infty}(a_n + b_n) = +\infty$$

(4) Suppose $0 < a < b$. Define $a_0 = a$, $b_0 = b$, and
$$a_{n+1} = \sqrt{a_n b_n} \qquad b_{n+1} = \frac{1}{2}(a_n + b_n)$$
for every $n \in \mathbb{N}$.
   (a) Prove that $(a_n)_{n=1}^{\infty}$ is an increasing sequence.
   (b) Prove that $(b_n)_{n=1}^{\infty}$ is an decreasing sequence.
   (c) Show that both sequences converge and that their limits are equal.

   Note: We call a_{n+1} the geometric mean of a_n and b_n and call b_{n+1} the arithmetic mean of a_n and b_n.