Homework 4 for M365C

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Wednesday February 23rd at 10 am

(1) Suppose $(a_n)_{n=1}^\infty$ and $(b_n)_{n=1}^\infty$ are sequences in $\mathbb{R}$ (not necessarily bounded). Show that
   (a) $\limsup_{n\to\infty}(-a_n) = -\liminf a_n$
   (b) $\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$.
   (c) $\limsup_{n\to\infty} ca_n = c \limsup_{n\to\infty} a_n$ where $c > 0$.

(2) Show that if $(a_n)_{n=1}^\infty$ is a convergent sequence in $\mathbb{R}$ then $(a_n)_{n=1}^\infty$ is Cauchy.

(3) Show that if $(a_n)_{n=1}^\infty$ is Cauchy and $(b_n)_{n=1}^\infty$ is a convergent subsequence of $(a_n)_{n=1}^\infty$ then $(a_n)_{n=1}^\infty$ is convergent and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$. Thus conclude a Cauchy sequence is convergent.

(4) Show that the harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges by showing that $s_{2n} - s_n > 1/2$.