

EXAM 1 FOR M427K - FEBRUARY 21ST

- You have 50 minutes.
- Please use the provided scratch paper.
- Calculators are *NOT* allowed.

- (1) Initially a tank contains 50 liters of water in which 1000 grams of silver chloride are dissolved. Solution at a concentration of 10 grams per liter enters at a rate of 5 liters per minute and the well mixed solution flows out at the same rate.
- (a) Set up an initial value problem for the quantity of silver chloride dissolved in the tank at time t .
 - (b) Solve the initial value problem.
 - (c) What is the limiting quantity of silver chloride dissolved in the tank as $t \rightarrow \infty$?
- (2) Solve the ordinary differential equation

$$\frac{dy}{dx} = \frac{x^2 y}{1 + x^3}$$

You do not need to find the solution explicitly.

- (3) Consider the ordinary differential equation

$$(2x^3 + 3y)dx + (3x + y - 1)dy = 0.$$

- (a) Show that it is exact.
 - (b) Solve the equation. You do not need to find the solution explicitly.
- (4) Consider the autonomous ordinary differential equation

$$\frac{dy}{dt} = f(y) \quad \text{where } f(y) = y^3(y^2 - 4)(y - 1)^2.$$

- (a) Sketch $f(y)$ vs y .
 - (b) Find the equilibrium solutions and classify each according to its stability.
 - (c) Sketch $y(t)$ vs t .
- (5) Solve the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 7, \quad y'(0) = -6.$$