Math 427K Exam 2

Name:	_____
UT EID:	_____

Instructions

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 10 points.
- You have 50 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

Method of Undetermined Coefficients Table

$g(t)$	$Y(t)$
$P_n(t) = a_n t^n + a_{n-1}t^{n-1} + \cdots + a_0$	$t^s(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)$
$P_n(t)e^{\alpha t}$	$t^s(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)e^{\alpha t}$
$P_n(t)e^{\alpha t}\begin{cases}\sin(\beta t)\\ \cos(\beta t)\end{cases}$	$t^s\big[(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)e^{\alpha t}\cos(\beta t) + (B_n t^n + B_{n-1}t^{n-1} + \cdots + B_0)e^{\alpha t}\sin(\beta t)\big]$

For Instructor's Use

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Total	_____

Question 1 [10 Points]

Find the general solution to the second order linear dordinary ifferential equation

$$(x-1)y''(x) - xy'(x) + y(x) = 0$$

given that $y_1(x) = e^x$ is a solution.

Question 2

[6 Points] Find the general solution to the inhomogeneous second order constant coefficient ordinary differential equation

$$y''(t) + 6y'(t) + 9y(t) = 125te^{2t}.$$

[4 points] Give the **form** of a particular solution to the inhomogeneous second order constant coefficient ordinary differential equation

$$y''(t) + 9y(t) = 3\sin(t).$$

You do **not** need to find the values of the undetermined coefficients.

Question 3 [10 points]

Find the general solution to the inhomogeneous second order linear ordinary differential equation

$$t^2y''(t) + ty'(t) - 4y(t) = t^4$$

given that $y_1 = t^2$ and $y_2 = t^{-2}$ are solutions of the associated homogeneous ordinary differential equation

$$t^2y''(t) + ty'(t) - 4y(t) = 0$$

Question 4

[8 Points] Assuming that the ordinary differential equation

$$(1 - x^2)y''(x) + y(x) = 0$$

has a solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$ determine the recurrence rules for the coefficients a_n. You do **not** need to find the solution of the recurrence rules.

[2 Points] Give a lower bound for the radius of convergence of the series solution to

$$(1 - x^2)y''(x) + y(x) = 0$$

and explain.