

MATH 427K EXAM 3

Name: _____
UT EID: _____

INSTRUCTIONS

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 10 points.
- You have 50 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

FOR INSTRUCTOR'S USE

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
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Total	_____

Question 1 [10 Points]

Solve the initial value problem

$$y''(t) + 4y'(t) + 4y(t) = h(t), \quad y(0) = 0, \quad y'(0) = 1$$

for the discontinuous forcing function

$$h(t) = \begin{cases} 0 & 0 \leq t < 2 \\ e^{-3t} & 2 \leq t \end{cases}$$

using the Laplace transform.

Question 2

(a) [5 points] Find the inverse Laplace transform of

$$\frac{2s}{(s^2 + 1)^2}.$$

(b) [5 points] Solve the initial value problem

$$y''(t) + y(t) = \delta(t - \pi) - \delta(t - 3\pi), \quad y(0) = 0, \quad y'(0) = 0$$

using the Laplace transform. Sketch the solution $y(t)$.

Question 3

Consider the conduction of heat in a 50 cm iron rod ($\alpha^2 = 0.12 \text{ cm}^2/\text{sec}^2$). Suppose the left hand end of the bar is maintained at a constant temperature of 10°C and the right hand end point is maintained at a constant temperature of 35°C . Let $u(x, t)$ denote the temperature of the bar at position x ($^\circ\text{C}$) at time t

[2 Points] Give the partial differential equation, initial and boundary values that govern this problem.

[2 Points] Find the steady state.

[2 Points] Give the general form of the solution for $u(x, t)$ as a combination of basic solutions.

[2 Points] Give the formula to compute the coefficients from the initial temperature distribution $f(x)$.

[2 points] Give the asymptotic behaviour of the temperature as $t \rightarrow \infty$.

Question 4

Consider a 30cm long vibrating string. Suppose the vertical displacement of the string, denoted $u(x, t)$, satisfies the wave equation

$$4u_{xx} = u_{tt}$$

with the conditions

$$u(0, t) = u(30, t) = 0$$

$$u(x, 0) = 0$$

$$u_t(x, 0) = g(x)$$

(a) [4 Points] Give the general form of the solution for $u(x, t)$ as a combination of basic solutions.

(b) [2 Points] Give the formula to compute the coefficients from the initial velocity $g(x)$.

(c) [4 Points] Given

$$g(x) = \sin(2\pi x)$$

find the solution $u(x, t)$.