

EXAM 1 FOR M325K

Name: _____
UT EID: _____

Complete ALL Problems. You have 75 minutes.

(1) (a) Define what it means for two logical expressions, P and Q , to be *logically equivalent*.

(b) Give an explanation why if $P \equiv P'$ and $Q \equiv Q'$ then

$$P \wedge Q \equiv P' \wedge Q'.$$

(c) Verify the logical equivalence

$$p \Rightarrow (q \vee r) \equiv (p \wedge \neg q) \Rightarrow r$$

by completing the truth table below.

p	q	r	
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

(2) (a) Give the definition of $p \Rightarrow q$ using the basic operators \neg , \vee , and \wedge .

(b) Define the *converse* and *contrapositive* of a conditional statement $p \Rightarrow q$.

(c) Verify the *Double Negation Law*

$$\neg\neg p \equiv p$$

using a chain of logical equivalences derived from the 5 basic laws of propositional calculus. State each law that you use.

(3) (a) Define what it means for a *logical expression* to be a *tautology*.

(b) Define what it means for an *argument* to be *valid*.

(c) Write the argument

$$\begin{aligned} & p \vee q \\ & p \Rightarrow \neg q \\ & p \Rightarrow r \\ \therefore & r \end{aligned}$$

in propositional form.

(d) Complete the truth table below for the propositional form of the argument.

p	q	r	
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

(e) Is the argument valid or invalid ? Why ?

(4) (a) Write the argument

$$\begin{array}{l} p \vee q \\ q \Rightarrow r \\ (p \wedge s) \Rightarrow t \\ \neg r \\ \neg q \Rightarrow (u \wedge s) \\ \therefore t \end{array}$$

in propositional form.

(b) Test it for validity using the alternative test for validity.

(5) Consider the expression P defined by

p	q	r	s	P
1	1	1	1	1
1	1	1	0	0
1	1	0	1	1
1	1	0	0	0
1	0	1	1	1
1	0	1	0	0
1	0	0	1	1
1	0	0	0	1
0	1	1	1	0
0	1	1	0	1
0	1	0	1	0
0	1	0	0	1
0	0	1	1	0
0	0	1	0	1
0	0	0	1	0
0	0	0	0	0

(a) Write a disjunctive normal form for P .

(b) Write a conjunctive normal form for P .

(c) Write an optimized disjunctive normal form for P by completing the Karnaugh map

(6) Consider the model

Objects	True Propositions
a	$P(a) \quad Q(a)$
b	$Q(b) \quad R(b)$
c	$P(c) \quad R(c)$

For each of the following propositions determine its truth value in the model. Give an explanation.

(a) $\exists y (\forall x (P(x) \Rightarrow Q(y)))$

(b) $\forall x (Q(x) \Rightarrow R(x))$

(c) $\exists x (P(x) \Rightarrow (Q(x) \wedge R(x)))$

(d) $(\forall x (P(x) \vee Q(x))) \Rightarrow (\exists x \neg R(x))$

(e) $\forall x (\exists y ((P(x) \wedge Q(y)) \Rightarrow R(x)))$

(7) Test

$$\begin{aligned} & \exists x (P(x) \vee Q(x)) \\ & \forall x (P(x) \Rightarrow R(x)) \\ & \forall x (Q(x) \Rightarrow S(x)) \\ \therefore & \exists x (R(x) \vee S(x)) \end{aligned}$$

for validity. Clearly indicate the contradiction or give a model which is a counterexample.

(8) Test

$$\begin{aligned} & \forall x \left((P(x) \vee Q(x)) \wedge \neg(P(x) \wedge Q(x)) \right) \\ & \forall x (P(x) \Rightarrow R(x)) \\ & \forall x (Q(x) \Rightarrow S(x)) \\ \therefore & \forall x \left((R(x) \vee S(x)) \wedge \neg(R(x) \wedge S(x)) \right) \end{aligned}$$

for validity. Clearly indicate the contradiction or give a model which is a counterexample.