Exam 2 for M325K

Name:	_____
UT EID:	_____

Instructions

- Please put your name and UT EID in the space provided.
- There are 6 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.

For Instructor's Use

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Question 5	_____
Question 6	_____
Total	_____

(1) (a) Prove that for all sets A, B

$$\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$$

(b) Find a counterexample to the statement that for all set A, B, and C

$$(A \cup B) \cap C = A \cup (B \cap C).$$

(2) (a) Using our main theorem on the subset relation prove:

For all sets $A, B \subseteq U$, if $A \subseteq B$ and $B \subseteq C^c$ then $A \cap C = \varnothing$.

(b) Recall that $A \triangle B = (B \backslash A) \cup (A \backslash B)$. Prove :

If $A \triangle B \subseteq B$ then $A \subseteq B$.

[Hint: Prove the contrapositive]

(3) Prove that for all sets A, B, C

$$(A \backslash B) \backslash C = A \backslash (B \cup C).$$

using the element method.

(4) (a) Give the formula for the number of r element subsets taken from a set of n elements.

(b) Give the formula for the number of permutations of a collection of n objects made up of k types with n_i indistinguishable objects of type i for $1 \leq i \leq k$.

(c) Give the formula for the number of r-element mutisets (r-combinations with repitition) from a set with n elements.

(d) Explain how to obtain the formula in (c) from the from the formula in (b).

(5) (a) Give the number of solutions to

$$x_1 + x_2 + x_3 = 9$$

if each $x_i \geq 1$ is a positive integer. (Order matters $x_1 = 1$ and $x_2 = x_3 = 4$ is a different solution to $x_2 = 1$ and $x_1 = x_3 = 4$.)

(b) Give the number of ways of reordering the letters in

SEVEN SAMURAI

ignoring the space.

(c) Give the number of ways of reordering the letters in

SEVEN SAMURAI

including the space so that we end up with 2 words each of which has at least 3 letters.

(6) An instructor gives an exam with 10 questions, 6 of which involve proofs, and 4 of which do not.

(a) Suppose students only need to complete 6 questions. How many different choices of 6 questions are possible ?

(b) Suppose that the 6 questions must be made up of 4 that require proofs and 2 that do not require proofs. How many different choices of 6 questions are possible ?

(c) Suppose that the instructor rules that questions 1 and 2 are too similar and at most one of them can be part of the 6 questions. If this is the only restriction how many different choices of 6 questions are possible ?