Homework 1 for M325K

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday July 18th

(1) Verify the logical equivalences
   (a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.
   (b) $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$.
   using truth tables.
(2) Prove the Idempotent Law

$$p \equiv p \wedge p$$

   using a chain of equivalences derived from the first 5 laws from Theorem 1.1.1
(3) For each of the following conditional propositions give the negation, the converse, and the contrapositive.
   (a) If n is divisible by 6 then n is divisible by 3. Here n is a specified integer.
   (b) If John is a computer scientist then John is shy.
(4) Test for validity the Law of Division into Cases

$$\begin{aligned} & p \vee q \\ & p \Rightarrow r \\ & q \Rightarrow r \\ \therefore\ & r \end{aligned}$$

   using the alternative test for validity.

(5) Consider the logical argument.

I plant peas or I plant beans
If I planted peas last year then I don't plant peas
If I planted beans last year then I don't plant beans
If I plant beans then I don't plant squash
I planted beans last year
If I don't plant beans then I plant squash or I plant melon
I don't plant both squash and melon
$\therefore$ I plant peas
I plant squash or melon but not both
Last year I planted beans but not peas.

(a) Define propositional variables and write out the argument in logical form.
(b) Give the single logical expression corresponding to the above argument.
(c) Test the argument for validity.

(6) Consider the following truth table.

p	q	r	s	
1	1	1	1	1
1	1	1	0	0
1	1	0	1	1
1	1	0	0	0
1	0	1	1	1
1	0	1	0	0
1	0	0	1	1
1	0	0	0	0
0	1	1	1	0
0	1	1	0	0
0	1	0	1	0
0	1	0	0	1
0	0	1	1	0
0	0	1	0	1
0	0	0	1	0
0	0	0	0	1

(a) Write a disjunctive normal form for the truth table.
(b) Write a conjunctive normal form for the truth table.
(c) Write an optimized disjunctive normal form for the truth table using a Karnaugh map.

(7) Define the logical operator | read NAND by

p	q	$p\|q$
1	1	0
1	0	1
0	1	1
0	0	1

Prove that *any* logical expression can be written using *only* the | operator.

Hint: Our normal form procedures show you can write *any* logical expression using only $\neg$, $\wedge$, and $\vee$. Thus it suffices to show you can write $\neg$, $\wedge$, and $\vee$ using |.