

HOMEWORK 2 FOR M325K

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE MONDAY JULY 25TH

(1) A menagerie consists of

- 7 brown dogs
- 2 black dogs
- 6 grey cats
- 10 black cats
- 5 blue birds
- 6 yellow birds
- 1 black bird

Consider the statements

- (a) There is an animal in the menagerie that is red.
- (b) Every animal in the menagerie is a mammal or a bird.
- (c) Every animal in the menagerie is brown, black, or grey.
- (d) There is an animal in the menagerie that is neither a cat nor a dog.
- (e) No animal in the menagerie is blue.
- (f) There is in the menagerie a dog, a cat, and a bird that are the same colour.

For each of the statements

- (a) Write a closed predicate-calculus form.
- (b) Determine the truth value of the statement.

(2) For each of the following expressions write the expression in negation normal form

- (a) $\neg((\forall x P(x)) \Rightarrow (\exists y Q(y)))$.
- (b) $\neg((\forall x (\forall y (P(x) \wedge Q(x)))) \wedge (\exists x (\exists y (P(x) \vee Q(x)))))$.
- (c) $\neg(\forall \epsilon > 0 (\exists \delta > 0 (\forall x (\forall y |x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon)))$.

(3) (a) All discrete mathematics students can tell a valid argument from an invalid one.

All thoughtful people can tell a valid argument from an invalid one.

\therefore All discrete mathematics students are thoughtful people.

- (b) Consider the arguments
- If I complete an argument without complaint then it is one I understand
 - This argument is not arranged in the usual order
 - Easy examples do not make my head ache
 - I cannot understand arguments that are not in the usual order
 - I never complain about an argument unless it gives me a headache
- \therefore This argument is not easy.

For each argument

- (i) Write the argument as a single closed predicate calculus form.
- (ii) Test the argument for validity.

(4) Let $A = \{1, 3, 5, 7, 9, 11, 13\}$, $B = \{2, 3, 5, 7, 11, 13\}$, $C = \{2, 4, 6, 8, 10, 12\}$, and $D = \{1, 2, 3, 6\}$.

Find

- (a) $A \cup B$.
 - (b) $A \cap B$.
 - (c) $A \cup C$.
 - (d) $A \cap C$.
 - (e) $A \setminus B$.
 - (f) $B \setminus A$.
 - (g) $B \cup C$.
 - (h) $B \cap C$.
 - (i) $\mathcal{P}(D)$.
- (5) For each of the following draw a Venn diagram for sets A , B , and C that satisfy the given conditions
- (a) $C \subseteq A$, $A \cap C = \emptyset$
 - (b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$.
 - (c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$, and $C \not\subseteq B$.