

HOMEWORK 3 FOR M325K

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE MONDAY AUGUST 1ST

(1) Let $A = \{\alpha, \beta\}$ and $B = \{a, b\}$. Find

- $A \times B$
- $\prod_{i=1}^3 A$
- $(A \times A) \times B$
- $A \times (A \times B)$
- $\mathcal{P}(A \times A)$

(2) (a) Prove the Idempotent Law,

For any set A , $A \cup A = A$.

using the first 5 laws.

(b) Prove the Laws of Universal Bounds;

For any $A \subseteq U$, $A \cap \emptyset = \emptyset$ and $A \cup U = U$.

using the first 5 laws and the Idempotent Law proved above.

(3) Prove:

Theorem. Let $A \subset U$ be a set. If a set $B \subset U$ satisfies

(a) $A \cap B = \emptyset$.

(b) $A \cup B = U$.

then $B = A^c$

(4) Using the last theorem and the Laws of Universal Bounds prove the De Morgan's Law

$$(A \cup B)^c = A^c \cap B^c.$$

(5) Prove:

Corollary. For any sets A and B we have $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

as a corollary of our main theorem on the subset relation.

(6) Prove

Theorem. If $A \subseteq U$ is a set and $B \subseteq U$ is the largest set disjoint from A in U i.e. B satisfies

(a) $A \cap B = \emptyset$.

(b) If $C \subseteq U$ and $A \cap C = \emptyset$ then $C \subseteq B$.

then $B = A^c$.

using the previous theorem and our main theorem on the subset relation.

(7) Prove

Theorem. If A and B are sets and C is the smallest set that contains A and B i.e. C satisfies

(a) $A \subseteq C$ and $B \subseteq C$.

(b) If $A \subseteq D$ and $B \subseteq D$ then $C \subseteq D$.

then $C = A \cup B$.

using our main theorem on the subset relation.