Homework 3 for M325K

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

Due Monday August 8th

(1) A coin is tossed 8 times. Each time a head or a tail is produced.
   (a) How many possible sequences of heads and tails are possible ?
   (b) How many sequences are there with exactly 3 tails ?
   (c) Find a formula for the number of sequences with exactly k tails where k is an integer with $0 \leq k \leq 8$.

(2) Hexadecimal numbers are represented by the sixteen "digits" 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. A 4 digit number is a string of 4 "digits" with the proviso that the leftmost digit is not 0.
   (a) How many different 4 digit hexadecimal numbers are there ?
   (b) How many different 4 digit hexadecimal numbers are there that have no repeated digits ?
   (c) How many different 4 digit numbers are constructed from 3 distinct digits with one of them repeated exactly twice ?

(3) Consider a sequence of length n made from the symbols $\{a, b, c, d\}$.
   (a) How many sequences are there with at least one pair of adjacent characters the same ?
   (b) How many sequences are there with exactly one pair of adjacent characters the same ?

(4) Consider the word $NEWZEALAND$.
   (a) How many distinct ways can you reorder the letters ignoring the space ?
   (b) How many distinct ways can you reorder the letters including the space so that you get two "words" of non-zero length ?
   (c) How many distinct ways can you reorder the letters including the space so that you get two "words" each of which contains at least one vowel ? Careful not to double count !

(5) A health food store stocks 8 different types of "complete breakfast foods" each of which consists of brightly coloured shapes which do not occur in nature.
   (a) How many ways can the inventory of 50 boxes be distributed among the 8 different types ?
   (b) How many ways can the inventory of 50 boxes be distributed among the 8 different types if there must be at least 5 boxes of each type ?

(6) How many ways are there of writing 8 as a sum of (not necessarily distinct) positive integers ?