

HOMEWORK 5 FOR M325K

- Please label your homework clearly with your name.
- Homework must be neatly written on one side of the paper only and should be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE FRIDAY AUGUST 12TH

- (1) Prove using regular induction that $7^n - 1$ is divisible by 6 for all integers $n \geq 1$.
- (2) Prove using strong induction that sequence $(b_n)_{n=1}^{\infty}$ defined by

$$b_1 = 4, \quad b_2 = 8, \quad b_k = b_{k-1} + b_{k-2} \quad \text{for } k \geq 3$$

has every term, b_n , divisible by 4.

- (3) Show that the relation *congruence modulo 5* on the integers, defined by

$$a \equiv b \pmod{5} \text{ if } a - b \text{ is divisible by } 5,$$

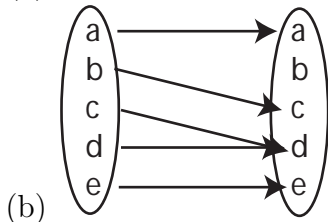
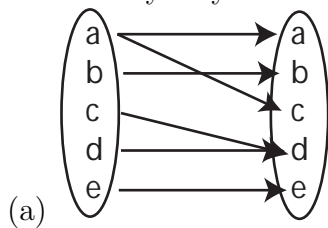
is an equivalence relation. What is the equivalence class of 3 ?

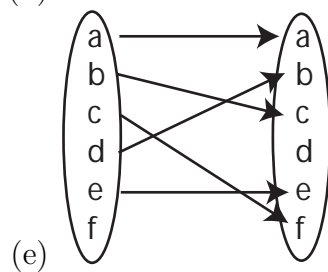
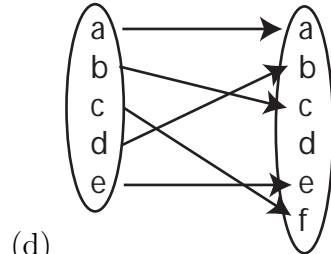
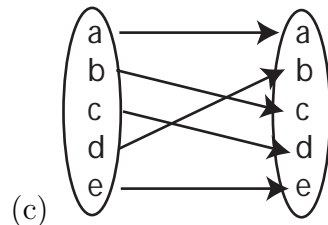
- (4) Let $P[x]$ be the set of polynomials in the single variable x . Show that the relation \equiv on $P[x]$ defined by

$$f \equiv g \text{ if } \frac{d^2 f}{dx^2} = \frac{d^2 g}{dx^2}$$

is an equivalence relation. What is the equivalence class of 0?

- (5) Draw the Hasse diagram for the partial order of $\mathcal{P}(\{a, b, c\})$ by the relation $A \preceq B$ if $A \subseteq B$.
- (6) Draw the Hasse diagram for the partial order of $\{1, \dots, 15\}$ by the partial relation $a \preceq b$ if a divides b .
- (7) For each of the following relations determine if it is a function. If it is a function say whether it is one-to-one, onto, and give its range. If it is not a function say why





(8) For each of the following functions say whether it is one-to-one, whether it is onto, and give its range.

- (a) $f(x) = x^2$ as a function from the integers to the non-negative integers.
 (b) $f(x) = |x|$ as a function from the integers to the non-negative integers.
 (c) $f(x) = 4x$ as a function from the integers to the integers.