QUIZ 3 FOR M325K

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| Name: | |
| UT EID: | SOLUTION |

(1) Consider the predicate calculus expression

$$(\forall x\ (\exists y\ (P(x) \wedge Q(y)))) \Rightarrow (\exists y\ (P(y) \wedge Q(y))).$$

(a) Define what it means for a well-formed predicate calculus formula to be closed. Is this expression closed?

An expression is closed if all predicate variables are bound. A variable $x$ is bound if it occurs within the scope of either a $\exists x$ or a $\forall x$

(b) Write this expression in negation normal form.

$$(\forall x\ (\exists y\ (P(x) \wedge Q(y)))) \Rightarrow (\exists y\ (P(y) \wedge Q(y)))$$
$$\equiv \neg(\forall x (\exists y\ (P(x) \wedge Q(y)))) \vee (\exists y\ (P(y) \wedge Q(y)))$$
$$\equiv (\exists x\ (\forall y\ (\neg P(x) \vee \neg Q(y)))) \vee (\exists y\ (P(y) \wedge Q(y)))$$

This is in negation normal form

(2) Consider the model

| Objects | True Statements | |
|---:|---|---|
| $a$ | $P(a)$ | $Q(a)$ |
| $b$ | $Q(b)$ | $R(b)$ |
| $c$ | $R(c)$ | |

For each of the following closed well-formed predicate calculus expressions determine its truth value in the given model.

(a) $\exists x\ ((P(x) \Rightarrow Q(x)) \wedge R(x)) \equiv ((P(a) \Rightarrow Q(a)) \wedge R(a)) \vee ((P(b) \Rightarrow Q(b)) \wedge R(b)) \vee ((P(c) \Rightarrow Q(c)) \wedge R(c))$

$(P(b) \Rightarrow Q(b)) \wedge R(b)$ is true so the $\exists$ is true.
In fact $(P(c) \Rightarrow Q(c)) \wedge R(c)$ is also true.

(b) $\forall x\ (Q(x) \vee R(x)) \equiv (Q(a) \vee R(a)) \wedge (Q(b) \vee R(b)) \wedge (Q(c) \vee R(c))$

This is true since $Q(a)$, $Q(b)$, and $R(c)$ are true.

(c) $(\exists x\ (P(x) \Rightarrow R(x))) \equiv (P(a) \Rightarrow R(a)) \vee (P(b) \Rightarrow R(b)) \vee (P(c) \Rightarrow R(c))$

which is true since both $P(b) \Rightarrow R(b)$ and $P(c) \Rightarrow R(c)$ are true.

(d) $((\exists x\ (P(x) \Rightarrow Q(x))) \wedge (\exists x\ (Q(x) \Rightarrow R(x)))) \Rightarrow (\exists x\ (P(x) \Rightarrow R(x)))$

True since by (c) the conclusion of the implication is true. The truth value of the premise is irrelevant.
In fact the premise are also true.
This is the propositional form of an invalid argument, there are models in which it is false (replace $R(c)$ by $P(c)$).