

Test the following arguments for validity. Clearly identify the contradiction or give a model that is a counterexample.

$$(1) \left(\left(\forall x (P(x) \Rightarrow Q(x)) \right) \wedge \left(\exists x \neg Q(x) \right) \right) \Rightarrow \left(\exists x \neg P(x) \right)$$

for all $\forall x (P(x) \Rightarrow Q(x))$ $\forall x P(x)$.

objects

a
 $\neg Q(a)$.

$P(a)$

$P(a) \Rightarrow Q(a)$

We reach a contradiction. We have $\neg Q(a)$ and $P(a)$ but then $P(a) \Rightarrow Q(a)$ is false.

$$(2) \left(\left(\forall x ((P(x) \wedge Q(x)) \Rightarrow R(x)) \right) \wedge \left(\exists x P(x) \right) \wedge \left(\exists x Q(x) \right) \right) \Rightarrow \left(\exists x R(x) \right)$$

For all $\forall x ((P(x) \wedge Q(x)) \Rightarrow R(x))$ $\forall x \neg R(x)$

Object

a

b

$P(a)$

$Q(b)$

$(P(a) \wedge Q(a)) \Rightarrow R(a)$

$(P(b) \wedge Q(b)) \Rightarrow R(b)$

$R(a)$

$\neg R(b)$

Following the logical implications

$\neg (P(a) \wedge Q(a))$

$\neg (P(b) \wedge Q(b))$

$\neg Q(a)$

$\neg P(b)$

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objects

a
 $\neg Q(a)$.

$P(a)$

$P(a) \Rightarrow Q(a)$

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For all $\forall x ((P(x) \wedge Q(x)) \Rightarrow R(x))$ $\forall x \neg R(x)$

Object

a

b

$P(a)$

$Q(b)$

$(P(a) \wedge Q(a)) \Rightarrow R(a)$

$(P(b) \wedge Q(b)) \Rightarrow R(b)$

$R(a)$

$\neg R(b)$

Following the logical implications

$\neg (P(a) \wedge Q(a))$

$\neg (P(b) \wedge Q(b))$

$\neg Q(a)$

$\neg P(b)$