UT EID: SOLUTIONS

You may use a calculator otherwise you may give the solutions using factorials.

(1) Prove that

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

by induction on n. You may take the base case to be $n=1$.

Let $P(n)$ be the proposition $1+2+\cdots+n=\frac{n(n+1)}{2}$. $1=\frac{1\times(1+1)}{2}$ so $P(1)$ is true. This establishes the base case.

Now we have to show if $P(n)$ is true then $P(n+1)$ is true. Assume $P(n)$ is true (induction hypothesis).

$$\begin{aligned} 1+2+\cdots+n+(n+1) &= (1+2+\cdots+n)+(n+1) = \frac{n(n+1)}{2}+n+1 \quad \text{by ind. hyp.} \\ &= \frac{n(n+1)}{2}+\frac{2(n+1)}{2} = \frac{(n+2)(n+1)}{2} \\ &= \frac{(n+1)((n+1)+1)}{2} \end{aligned}$$

Thus $P(n+1)$ is true. By the principle of induction $P(n)$ is true for all $n\geq 1$

(2) Give the 3 properties that a relation R on a set X must satisfy for R to be an *equivalence relation.*

Reflexive: For all $x\in X$ xRx.

Symmetric: For all $x,y\in X$ if xRy then yRx.

Transitive: For all $x,y,z\in X$ if xRy and yRz then xRz

(3) For each of the following functions say whether it is one-to-one, onto, or a bijection. If it is a bijection give a formula for its inverse.

(a) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x$.

Bijection. Inverse $f^{-1}(x)=\frac{1}{2}x$.

(b) $f:\mathbb{Z}\to\mathbb{Z}$ defined by $f(x)=2x$. ($\mathbb{Z}$ means integers)

One-to-one but not onto. (Range is all even integers)

(c) $f:\mathbb{Z}\to\{0,1,2,3,4\}$ defined by $f(x)$ is the remainder after x is divided by 5.

Onto but not one-to-one.