Math 361K Exam 1

Name:	____________________
UT EID:	____________________

Instructions

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

For Instructor's Use

Question 1	______
Question 2	______
Question 3	______
Question 4	______
Total	______

Problem 1

1. [2 Points] Define what it means for a sequence (a_n) to have limit L.

2. [2 Points] Prove that if the limit of a sequence (a_n) exists then it is unique

3. [6 Points] Using the definition of limit prove the limit law

   Theorem. *If (a_n) and (b_n) are convergent sequences then $(a_n + b_n)$ is a convergent sequence and*

$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n.$$

Problem 2

1. [2 Points] Define what it means for a sequence (a_n) to be *non-decreasing.*

2. [6 Points] Prove the following theorem

   Theorem. *If (a_n) is a non-decreasing sequence and (a_n) is bounded from above then (a_n) is convergent.*

3. [2 Points] Let (a_n) be a sequence in $\mathbb{R}$. Define what is meant by $\limsup_{n\to\infty} a_n$.

Problem 3

1. [5 Points] Prove the following theorem

   Theorem. *Let* (a_n) *be a sequence in* $\mathbb{R}$. *If*

   $$\limsup_{n\to\infty} a_n = A$$

   then for all $B > A$ *we have* $a_n < B$ *for all but finitely many* n.

2. [5 Points] Prove the following theorem

   Theorem. *Let* (a_n) *be a sequence in* $\mathbb{R}$. *If*

   $$\limsup_{n\to\infty} \sqrt[n]{|a_n|} < 1$$

   then $\lim_{n\to\infty} a_n = 0$

Problem 4

1. [3 Points] State the Nested Interval Property for $\mathbb{R}$. Identity clearly all the hypotheses and the conclusion.

2. [7 Points] Prove the Bolzano-Weierstrass theorem

   Theorem. *Every bounded sequence in* $\mathbb{R}$ *contains a convergent subsequence.*

   using the Nested Interval Property.

Problem 5

1. [3 Points] Define what it means for a sequence (a_n) to be a Cauchy sequence.

2. [7 Points] Prove the following theorem:

   Theorem. *If* (a_n) *is a convergent sequence then* (a_n) *is a Cauchy sequence.*