Math 361K Exam 2

Name:	____________________
UT EID:	____________________

Instructions

- Please put your name and UT EID in the space provided.
- There are 5 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

For Instructor's Use

Question 1	______
Question 2	______
Question 3	______
Question 4	______
Question 5	______
Total	______

Problem 1

1. [2 Points] Define what it means for a sequence $(a_n) \subseteq \mathbb{R}$ to be a **Cauchy sequence**.

2. [2 Points] Define what it means for a sequence $(a_n) \subseteq \mathbb{R}$ to be **bounded**.

3. [6 Points] Prove that every Cauchy sequence is bounded.

Problem 2

1. [4 Points] Define what it means for the infinite sum $\sum_{n=1}^{\infty} a_n$ to converge.

2. [6 Points] Prove the following theorem:

   Theorem. *If (a_n) is a sequence with $a_n \geq 0$ then either*

   (a) $\sum_{n=1}^{\infty} a_n$ *diverges to* $+\infty$*, or*

   (b) $\sum_{n=1}^{\infty} a_n$ *converges.*

Problem 3

1. [3 Points] Define what it means for an infinite sum $\sum_{n=1}^{\infty} a_n$ to **converge absolutely**.

2. [5 Points] Prove the following theorem:

   Theorem. *If the infinite sum* $\sum_{n=1}^{\infty} a_n$ *converges absolutely then* $\sum_{n=1}^{\infty} a_n$ *converges.*

3. [2 Points] Give an example of an infinite sum that is convergent but not absolutely convergent (i.e. that is conditionally convergent).

Problem 4

1. [5 Points] Prove the following theorem:

   Theorem. *If* $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *are convergent series and* $\alpha, \beta \in \mathbb{R}$ *then* $\sum_{n=1}^{\infty}(\alpha a_n + \beta_n)$ *is convergent and*

$$\sum_{n=1}^{\infty}(\alpha a_n + \beta_n) = \alpha\big(\sum_{n=1}^{\infty} a_n\big) + \beta\big(\sum_{n=1}^{\infty} b_n\big).$$

2. [5 Points]

Problem 5

1. [5 Points] Prove the following theorem:

   Theorem. *If* $\sum_{n=1}^{\infty} a_n$ *converges then* $\lim_{n\to\infty} a_n = 0$.

2. [5 Points] Give an example of an infinite series $\sum n = 1^{\infty} a_n$ that diverges but that has $\lim_{n\to\infty} a_n = 0$.