Math 361K Exam 3

Name:	______________________
UT EID:	______________________

Instructions

- Please put your name and UT EID in the space provided.
- There are 5 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

For Instructor's Use

Question 1	______
Question 2	______
Question 3	______
Question 4	______
Question 5	______
Total	______

Problem 1

1. [3 Points] Let $f : A \to \mathbb{R}$ and $c \in A$. Define what it means for f to be continuous at c using the ϵ and δ definition.

2. [7 Points] Prove:

   Theorem. *Let $f : A \to \mathbb{R}$ and $c \in A$. The function f is continuous at c if and only if for every sequence $(x_n) \subseteq A$ with $\lim_{n\to\infty} x_n = c$ we have $\lim_{n\to\infty} f(x_n) = f(c)$.*

   Hint:

Problem 2

1. [2 Points] Let $f : A \to \mathbb{R}$. Define what it means for f to be uniformly continuous.

2. [2 Points] Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous on $\mathbb{R}$ but not uniformly continuous on $\mathbb{R}$. *Explain.*

3. [6 Points] Prove:

   Theorem (Extreme Value Theorem). *If $f : [a, b] \to \mathbb{R}$ is continuous then there exists $c \in A$ such that $f(x) \le f(c)$ for all $x \in A$.*

   You may assume that we have already shown that f is bounded above. This is not related to uniform continuity.

Problem 3

1. [3 Points] Let $f : (a, b) \to \mathbb{R}$. Define what it means for f to be differentiable at $c \in (a, b)$.

2. [7 Points] Prove:

   Theorem. *If $f : (a, b) \to \mathbb{R}$ is differentiable at $c \in (a, b)$ and c is a* local maximum *then $f'(c) = 0$.*

Problem 4

1. [2 Points] Define what it means for $\pi = (x_i)_{i=0}^n$ to be a partition of $[a, b]$.

2. [2 Points] Define what it means for a partition π' to be a *refinement* of π.

3. [2 Points] Define what it means for $\sigma = (\xi_i)_{i=1}^n$ to be a sample associated to $\pi = (x_i)_{i=0}^n$.

4. [2 Points] Let $f : [a, b] \to \mathbb{R}$, $\pi = (x_i)_{i=0}^n$ a partition of $[a, b]$, and $\sigma = (\xi_i)_{i=1}^n$ a sample associated to π. Define $S(f, \pi, \sigma)$, the *Riemann sum* of f over the partition π with sample σ.

5. [2 Points] Let $f : [a, b] \to \mathbb{R}$. Define what it means for the number I to be the *Riemann integral* of f over $[a, b]$.

Problem 5

1. [5 Points] Suppose that I_1 and I_2 are Riemann integrals of f over $[a, b]$. Show that $I_1 = I_2$.

2. [5 Points] Prove

   Theorem. *Let $f : [a, b] \to \mathbb{R}$. If f is Riemann integrable over $[a, b]$ then f is bounded on $[a, b]$.*

   Hint: Prove the contrapositive