

## HOMEWORK 10 FOR M361K

- Please label your homework clearly with your name.
- Homework must be neatly written and must be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE TUESDAY MAY 2ND

- (1) (a) Let  $\pi = (x_i)_{i=0}^n$  be a partition of  $[-a, a]$ . Show that there exists a partition  $\pi' = (y_j)_{j=0}^{2m}$  with the properties
- Symmetry:** for all  $0 \leq j \leq 2m$   $y_j = -y_{2m-j}$ .
- Refinement:**  $\pi \leq \pi'$ .
- (b) Suppose  $f : [-a, a] \rightarrow \mathbb{R}$  is odd, i.e.  $f(-x) = -f(x)$ . Suppose that  $\int_{-a}^a f$  exists. Show that  $\int_{-a}^a f = 0$  using the definition of the Riemann integral.
- (c) Suppose  $f : [-a, a] \rightarrow \mathbb{R}$  is even, i.e.  $f(-x) = f(x)$ . Suppose that  $\int_{-a}^a f$  exists. Show that  $\int_{-a}^a f = 2 \int_0^a f$  using the definition of the Riemann integral.
- (2) Show that for  $f : [a, b] \rightarrow \mathbb{R}$ ,  $g : [a, b] \rightarrow \mathbb{R}$ , and  $\alpha, \beta \in \mathbb{R}$  we have

$$S(\alpha f + \beta g, \pi, \sigma) = \alpha S(f, \pi, \sigma) + \beta S(g, \pi, \sigma)$$

for all partitions  $\pi$  of  $[a, b]$  and all samples associated to  $\pi$ . Hence show that if  $f$  and  $g$  are Riemann integrable on  $[a, b]$  then  $\alpha f + \beta g$  is Riemann integrable and

$$\int_a^b (\alpha f + \beta g) = \alpha \int_a^b f + \beta \int_a^b g$$

- (3) Each of the following functions is defined on  $(0, 1]$ . Extend each to a function  $f : [0, 1] \rightarrow \mathbb{R}$  by setting  $f(0) = 0$ . For each function decide whether it is Riemann integrable on  $[0, 1]$  and give a proof.
- $\sin^2\left(\frac{1}{x}\right)$
  - $\frac{1}{x} \sin\left(\frac{1}{x}\right)$ .
  - $\ln(x)$ .
  - $\frac{\sin(x)}{x}$ .