

HOMWORK 2 FOR M361K

- Please label your homework clearly with your name.
- Homework must be neatly written and must be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE TUESDAY FEBRUARY 7TH

- (1) Show that

$$\lim_{n \rightarrow \infty} \frac{2n}{n^2 + 1} = 0$$

from the definition.

- (2) Show that

$$\lim_{n \rightarrow \infty} \frac{(-1)^n}{n} = 0$$

from the definition.

- (3) Prove from the definition that if (a_n) is a bounded sequence and (b_n) is a convergent sequence with $\lim_{n \rightarrow \infty} b_n = 0$ then $\lim_{n \rightarrow \infty} a_n b_n = 0$. Explain why we cannot use the product rule for limits.
- (4) Guess the limit of the sequence $a_n = \sqrt{n+2} - \sqrt{n}$ and prove it is the limit using the definition. Explain how you guessed the limit. *Hint: Use the procedure sometimes known as rationalizing.*
- (5) Prove that if (a_n) is a convergent sequence and (b_n) is a sequence that diverges to $+\infty$ then $(a_n + b_n)$ diverges to $+\infty$.
- (6) Prove that if (a_n) is a convergent sequence with $\lim_{n \rightarrow \infty} a_n > 0$ and (b_n) is a sequence that diverges to $+\infty$ then $(a_n b_n)$ diverges to $+\infty$.