

### HOMWORK 3 FOR M361K

- Please label your homework clearly with your name.
- Homework must be neatly written and must be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE TUESDAY FEBRUARY 14TH

- (1) A sequence  $(a_n)_{n=1}^{\infty}$  is called *bounded* if there exists  $M > 0$  such that  $|a_n| < M$  for all  $n \in \mathbb{N}$ .
  - (a) Prove that if  $(a_n)_{n=1}^{\infty}$  is a convergent sequence then it is bounded.
  - (b) Give an example of a bounded sequence  $(a_n)_{n=1}^{\infty}$  that is not convergent.
- (2) Find the periodic decimal expansion of  $\frac{22}{7}$  using the long division algorithm explained in class. This can take no more than 8 steps by the pigeonhole principle.

Archimedes of Syracuse proved that  $\frac{22}{7}$  is an upper bound for  $\pi$ . He also showed that  $\frac{223}{71}$  is a lower bound. Lambert showed that  $\pi$  was irrational in 1761 and Lindemann showed that  $\pi$  is in fact transcendental i.e. not the root of any polynomial with integer coefficients. When we do infinite series I will give some calculations for  $\pi$ .

- (3) The sum of two convergent sequences is convergent and the product of two convergent sequences is convergent. However products and sums of divergent sequences need not be divergent.
  - (a) Find two divergent sequences  $(a_n)_{n=1}^{\infty}$  and  $(b_n)_{n=1}^{\infty}$  such that their sum  $(a_n + b_n)_{n=1}^{\infty}$  is convergent.
  - (b) Find two divergent sequences  $(a_n)_{n=1}^{\infty}$  and  $(b_n)_{n=1}^{\infty}$  such that their product  $(a_n b_n)_{n=1}^{\infty}$  is convergent.
- (4) Prove that if  $(a_n)_{n=1}^{\infty}$  is convergent and  $(b_n)_{n=1}^{\infty}$  is divergent then  $(a_n + b_n)_{n=1}^{\infty}$  is divergent.
- (5) Find a counterexample to the statement  
If  $(a_n)_{n=1}^{\infty}$  is convergent and  $(b_n)_{n=1}^{\infty}$  is divergent then  $(a_n b_n)_{n=1}^{\infty}$  is divergent.

What restriction would you need to add to make the statement true ?