

HOMWORK 5 FOR M361K

- Please label your homework clearly with your name.
- Homework must be neatly written and must be stapled.
- Feel free to discuss your solutions with other students but try to solve the problems by yourself first.

DUE TUESDAY FEBRUARY 28TH

- (1) Provide an example of a sequence in \mathbb{Q} that is Cauchy in \mathbb{Q} but not convergent in \mathbb{Q} .
- (2) Let (a_n) be a sequence of real numbers. Show that if

$$\sum_{n=1}^{\infty} |a_{n+1} - a_n| < \infty$$

then (a_n) is a Cauchy sequence and hence convergent in \mathbb{R} .

Remark: We have not done infinite series in detail yet but the meaning should be clear.

- (3) A sequence (a_n) of real numbers is called “contractive” if there exists a $\lambda < 1$ such that for all $n \in \mathbb{N}$ we have

$$|a_{n+2} - a_{n+1}| \leq \lambda |a_{n+1} - a_n|.$$

Show that a contractive sequence is Cauchy and hence convergent in \mathbb{R} .

Remark: You can use the previous theorem or follow the proof using the direct formula for the sum of a geometric progression.