

MATH 427K EXAM 1

Name: _____
UT EID: _____

INSTRUCTIONS

- Please put your name and UT EID in the space provided.
- There are 4 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

FOR INSTRUCTOR'S USE

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Question 5	_____
Question 6	_____
Total	_____

Question 1 [10 Points]

A 1000L tank initially holds 200L of solution of $UO_2(NO_3)_2$ with an initial concentration of $2\mu g/L$. Solution containing $8\mu g/L$ of $UO_2(NO_3)_2$ salt enters the tank at the rate of $4L/min$. Well mixed solution is allowed to flow out of the tank at the rate of $2L/min$

1. [2 Points] Write an initial value problem describing the amount of $UO_2(NO_3)_2$ in solution after t minutes.
2. [5 Points] Solve the initial value problem to find the amount of $UO_2(NO_3)_2$ in solution after t minutes
3. [3 Points] Find the concentration of $UO_2(NO_3)_2$ in solution at the moment the tank is full. Compare this with the limiting concentration that would result if the tank had infinite volume.

Question 2 [10 Points]

1. [3 Points] Apply the Existence and Uniqueness theorem for first order linear ordinary differential equations to

$$ty' + y = t \sin(t), \quad y(\pi) = y_0$$

giving the largest interval for which solutions are guaranteed to exist and be unique.

2. [4 Points] Find the general solution of the ordinary differential equation

$$ty' + y = t \sin(t)$$

3. [3 Points] Find the value of y_0 such that the solution of the initial value problem

$$ty' + y = t \sin(t), \quad y(\pi) = y_0$$

exists for all times t . Explain what happens for other values of y_0 .

Question 3 [10 points]

1. [8 Points] Find an *explicit* solution to the ordinary differential equation

$$\frac{dy}{dt} = \frac{4t^3 + 1}{2y - 2}.$$

2. [2 Points] Find an *explicit* solution to the initial value problem

$$\frac{dy}{dt} = \frac{4t^3 + 1}{2y - 2}, \quad y(0) = -1.$$

Question 4 [10 points]

1. [5 Points] Solve the first order ordinary differential equation

$$(4x^3 + 3y) dx + (4y^3 + 3x) dy = 0.$$

You need not find the solution *explicitly*.

2. [5 Points] Find an integrating factor that makes the first order ordinary differential equation

$$(x - y^2) dx + 2xy dy = 0$$

into an exact equation. You do not need to solve the equation.

Question 5 [10 points]

Solve the homogeneous first order ordinary differential equation

$$\frac{dy}{dx} = \frac{x^3 + y^3}{3xy^2}.$$

You do not need to find the solution *explicitly*.

