Math 427K Exam 2

Name:	_____
UT EID:	_____

Instructions

- Please put your name and UT EID in the space provided.
- There are 6 questions each worth 10 points.
- You have 75 minutes to complete the test.
- Please write your working and solutions on the test paper. You may use the back of the pages.
- Calculators are not allowed.

Method of Undetermined Coefficients Table

$g(t)$	$Y(t)$
$P_n(t) = a_n t^n + a_{n-1}t^{n-1} + \cdots + a_0$	$t^s(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)$
$P_n(t)e^{\alpha t}$	$t^s(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)e^{\alpha t}$
$P_n(t)e^{\alpha t}\begin{cases}\sin(\beta t)\\ \cos(\beta t)\end{cases}$	$t^s\big[(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0)e^{\alpha t}\cos(\beta t) + (B_n t^n + B_{n-1}t^{n-1} + \cdots + B_0)e^{\alpha t}\sin(\beta t)\big]$

For Instructor's Use

Question 1	_____
Question 2	_____
Question 3	_____
Question 4	_____
Question 5	_____
Question 6	_____
Total	_____

Question 1

1. [3 Points] Find the general solutions of the homogeneous second order constant coefficient ordinary differential equation

$$y''(x) + y'(x) - 6y(x) = 0$$

2. [3 Points] Find the general solutions of the homogeneous second order constant coefficient ordinary differential equation

$$4y''(x) - 12y'(x) + 9y(x) = 0$$

3. [3 Points] Find the general solutions of the homogeneous second order constant coefficient ordinary differential equation

$$y''(x) + 2y'(x) + 5y(x) = 0$$

Question 2

1. [5 Points] Find the general solution to the inhomogeneous second order constant coefficient ordinary differential equation

$$y''(t) - 6y'(t) + 18y(t) = 45\cos(3t).$$

2. [5 points] Find the general solution to the inhomogeneous second order constant coefficient ordinary differential equation

$$y''(t) + 2y'(t) - 3y(t) = 4e^t + 5e^{2t}$$

Question 3 [10 Points]

Find the general solution to the second order linear ordinary differential equation

$$x^2y'' - x(x+2)y' + (x+2)y = 0$$

given that $y_1(x) = x$ is a solution.

Question 4 [10 points]

Find the general solution to the inhomogeneous second order linear ordinary differential equation

$$t^2y''(t) - 2y(t) = 3t^2 - 1$$

given that $y_1 = t^2$ and $y_2 = t^{-1}$ are solutions of the associated homogeneous ordinary differential equation

$$t^2y''(t) - 2y(t) = 0$$

Question 5 [10 Points]

Find a series solutions about $x_0 = 0$ to the second order ordinary differential equation

$$y'' - xy = 0$$

Find the recurrence relation and the first 4 terms in each of the two linearly independent solutions.

Question 6

1. [5 Points] Find a lower bound for the radius of convergence of series solutions about $x_0 = 0$ for the family of second order linear ordinary differential equations (the Chebyshev equations)

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0.$$

2. [5 Points] Let $\alpha = 2$ so that we obtain the Chebyshev equation

$$(1 - x^2)y'' - xy' + 4y = 0.$$

Show that this equation has a solution which is a second order polynomial.