Point Processes, Random Graphs and Stochastic Geometry

Graduate Course, 54640, M 393C

Fall 2017

Instructor

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Objectives

The course will be structured in 3 basic blocks

I. Point Processes,

II. Random Geometric Graphs,

III. Stochastic Geometry.

The instructional objective is that students having completed it be in a position:

- to start mathematical research on the domains in question;
- to use the described methodology in some applications, i.e. to develop new models coming from these applications, analyze and solve these models.

I. Point Processes

Point processes are a fundamental object in probability theory, at the same level of generality and use as e.g. second order stochastic processes. They were used as early as the first half of the 19-th century (Poisson) and are now ubiquitous in physics, computer science (image analysis, information theory, networks), engineering (electrical, material), life sciences (biology, ecology), earth science (seismology), sociology, etc. The objective of this block is that students master the basic formalism of point process on Euclidean spaces. The basic notions and tools to be covered are:

- Point processes as random measures;
- Stationarity of a point process;
- Marks of a point process;
- Moment measures;
- Palm calculus;
- Poisson point processes;
- Generating functionals.

II. Random Graphs

This block will discuss random graphs defined on point processes of the Euclidean space, which are commonly used in all the domains listed above. The main emphasis will be on the use of the methodology developed in I. to analyze random geometric graphs. The following notions will be covered:

- Random geometric graphs;
- Continuum percolation;
- Mass transport;
- Point maps and point shifts;
- Palm probability and mass transport.

III. Stochastic Geometry

Stochastic geometry is focused on the study of random geometric objects of e.g. the Euclidean space such as random sets, random tessellations. Kolmogorov is credited for having built the foundations of the field – the Boolean model and the Poisson-Voronoi tessellation – for analyzing the growth of crystals in materials. Nowadays it is also widely used in computer science and electrical engineering (image analysis, information theory, wireless communications), cosmology, hydrology, ecology, cell biology, to quote a few. The main objective of this block is that the students master the definition of the basic objects of stochastic geometry and the computation of their law. The basic notions and tools to be covered are:

- The Boolean model, particle processes and random closed sets;
- Processes of flats (line processes and hyperplane processes);
- Random tessellations (Voronoi, Delaunay);
- Shot noise fields.

References

The following material will be used in the course:

[DVJ] D. Daley & D. Vere-Jones, *Introduction to the Theory of Point Processes* Springer Verlag, second edition, 2008. pdf available through the math library.

[BBK] F. Baccelli, B. Blaszczyszyn and M. Karray, *Point Processes and Stochastic Geometry*, manuscript in preparation, 2017.

[SW] R. Schneider and W. Weil, *Stochastic and Integral Geometry*, Springer Verlag 2008.

[SKM] D. Stoyan, W. Kendall & J. Mecke, *Stochastic Geometry and its Applications*, John Wiley and Sons, second edition, 1995.

Grading

- Assignments: 1/3;
- 2 midterm exams: 1/3;
- One research paper to read and present (from a list of proposed papers): 1/3.

Practical Information

Class Hours: TBA

Office Hours: TBA

Course Policy: Course material will be available on Canvas.

Further Information

The following information should be taken into account:

UT Honor Code Information on the UT honor code can be found at http://catalog.utexas.edu/general-information/the-university/#universitycodeofconduct

Recommendations Regarding Emergency Evacuation Recommendations on emergency evacuation can be found at http://www.utexas.edu/safety/ or by calling Campus Safety and Security, 512-471-5767.