"You don't *really* understand a skill/concept until you teach it."

This simple piece of wisdom captures my teaching philosophy: the process that causes a person to learn *is* for him/her to attempt to take some collection of ideas and facts, synthesize and structure them, and articulate them to a target audience (namely, to teach). Designing activities so that a student's job is to exhibit these teaching behaviors is effective and rewarding.

The perspective of students-as-teachers affects my decisions when designing all of my courses. It has guided me when teaching Precalculus, Honors Discrete Mathematics, an individual conference course with an undergraduate (designed to help underrepresented minorities prepare for graduate school), an interdisciplinary First-Year seminar, and my department's course that trains our new graduate students to be TAs. This perspective on human learning even affects how I have led a few non-classroom activities including organizing a seminar that helps young graduate students give their first talk, structuring a learning seminar where the younger graduate students work through the core material for research in algebraic geometry, and acting as a sort of teaching sounding-board for other graduate students.

For me personally, this focus on students-as-teachers has led to receiving my department's Teaching Excellence award, being asked to redesign and teach Supervised Teaching in Mathematics, and having the deeply personal honor of former students regularly visiting to tell me that my class was the "best (and hardest!) class [they've] taken here at UT". I would like to make this abstract idea of students-as-teachers more specific by describing how I use it as the tool for instruction in two different classes.

**Precalculus:** I have been the instructor of record for a Precalculus class of 44 students. Usually, 10% of each Precalculus class drops the course within a month; I kept full enrollment for the whole semester. In fact, there were only 41 seats in my classroom, so I often had students sitting on the floor to be in my class. My focus on the students-as-teachers causes these students to learn powerful study and communication skills that allow them to succeed at (and enjoy) mathematics classes (sometimes for the first time in their lives). Traditionally, the students who take Precalculus at UT earn a C or lower when they take Calculus. Contrary to this dire prognosis, of the half of my class that did take Calculus, nearly all of them have come by my office to tell me how proud they are of their A or B in the first semester of Calculus.

One way in which I put my Precalculus students in the role of a teacher is to require each of them to give a 2-minute summary of a lecture at the beginning of one of our discussion sections. The student must prepare the summary on her own and practice giving it to me in office hours. Several of the students made a point of telling me after a test how they performed best on the ideas from the lecture that they had summarized and that they intended to do some version of the summarization process in *all* of their classes.

My students also learn study-skills in office hours because I require them to exhibit teacher-like behaviors before I will answer a question. I will not respond to a question like "I don't know how to do number 43 . . ." (not even a question!); instead the student must pick up a piece of chalk, show me what he tried, and attempt to tell me what is going wrong. Often, doing this on a clean chalkboard is enough for him to complete the problem correctly, and the rest of the time another student in the office hour is able to offer a suggestion that gets him back on track. Furthermore, once I have discussed a problem with a student, it becomes his responsibility to answer questions about that problem to all future visitors to the office hour.

My focus on students-as-teachers also leads to grading methods that develop study skills in students; the requirement is that their written work be sufficient to teach someone to do the problem, not just enough to get an answer. All problems must include some form of explanation or independent check for full credit, and an incorrect answer will only receive partial credit if it is well-organized and accompanied by an explanation. For example, knowledge of the graphs of basic functions and of graphing transformations is often tested by having the student match a list of functions to their graphs. On my tests, this problem also requires the student to explain why she is sure she has chosen the right pairings. At first, students find this hard, but by the second test they can put together a logical argument that approaches a proof. The tests also separate explaining general procedures from doing specific problems, which requires the student to be more reflective on her process. In addition, the students are often given the chance to correct their work for partial credit, which teaches them to look at the feedback they receive, make sense of it, and incorporate it into revisions.

I also choose groups for in-class work so that everyone may act as a teacher and practice communicating. For example, I often put students into groups of three including a strong student, a medium student, and a student who has been struggling. (These designations are fluid, sometimes changing during a given class period.) When these groups work on problems together, the strongest student usually leads while they work the problems, the medium student is then required to produce a well-organized, written version of the solution, and the weakest student is responsible for presenting the group's solution to the class. By adjusting the format of each student's work and its target audience, I can make the time more useful for all three students: each student is likely to succeed at a task tailored to challenge her, and she is focused on understanding instead of getting an answer. Moreover, the stricter audience of a peer helps the strong student keep working until her work is thorough, while the weaker student who explains an idea really takes ownership of it in a way that positively affects her confidence. This kind of structured group-work also multiplies the amount of feedback the students receive because I am no longer the only source. Eventually students work in groups outside of class and read their work aloud to make sure that it truly makes sense from the reader's perspective. Finally, this perspective shifts the students' goal from getting an answer to communicating their method.

None of my Precalculus students were mathematics majors, which presented me with other interesting contexts in which to have them communicate mathematics: applications to topics of personal interest. I assigned each larger-scale application problem to a particular student and helped him work on a short presentation (with a hand-out) of it to his peers. We took a few days out of the semester to give and discuss these presentations, which the students found illuminating. Also, I offered an extra-credit project called "Math in my Major", which many of my students took advantage of and enjoyed. They were asked to find an expert in their field of interest, articulate the skills learned in my course, ask for a concept/technique from the expert that can only be understood using these ideas, and write a 2-3 page paper explaining this application of Precalculus to their major. I learned many interesting ways in which the tools of Precalculus are used by pharmacists, nutritionists, chemists, and even sports medicine professionals.

**Discrete Mathematics:** I have also taught the Dean's Scholars Discrete Mathematics course twice with Dr. Michael Starbird. This honors class includes topics from introductory Graph Theory, Group Theory, Real Analysis, and Point-Set Topology (as such, the class is inappropriately named). We use an Inquiry-Based teaching method: the students work through a carefully designed mixture of computational and proof-based exercises to explore, and prove, the major theorems collectively.

The way we implement this teaching method requires students to model teaching behaviors in class. Class-time is filled with student presentations during which the presenter is acting as a teacher; more precisely, the presenter is teaching his *peers*, not the instructors. This distinction raises the bar significantly for the level of clarity required because it is much harder to teach a peer to understand your solution than it is to convince a mathematician that your solution is correct. Moreover, the presenter's peers direct their questions to the presenter so all questions must be carefully articulated. While the students model these teaching behaviors, everyone in the room is focused on structuring the presenter's ideas for themselves, which produces interesting conversations during class. Also, we often ask one of the peers to give a 2-3 sentence summary of a proof. The process of selecting the significant ideas from a proof and structuring them into a summary (another instance of students-as-teachers) leads to students' understanding the details of a proof and thinking about the structure of the proof as a whole. Furthermore, placing a student in the role of teacher, questioner, or summarizer forces him to have a reason for every sentence, which builds a personal understanding of the components of a complete argument. For example, I have moderated several fascinating discussions with groups of students about why we often, but not always, fix an $\varepsilon$ at the beginning of an analysis proof. This interactive use of class-time teaches students to talk about mathematics and convinces them that they have the ability to decide whether a proof or technique is correct instead of waiting for the instructor's verdict. Moreover, these roles combine the process that causes learning with the demonstration of understanding, so we can evaluate student learning without interrupting class with quizzes. This focus on students directly teaching each other culminates in the final projects at the end of the semester. In small groups, the students research topics that have become accessible (including Wall-Paper Groups, Solving the Rubik's Cube, and Computer Realizations of the Euler Circuit Theorem) and prepare a 25 minute presentation for their peers. The projects are graded, in large part, on their ability to have their peers do something interesting with the ideas that they have presented (in other words, the quality of the presentation as a teaching situation).

We encourage the focus on students-as-teachers outside of class as well by scoring the written homework on the clarity of the writing for a specific audience. The audience for the written homework is a mathematics major who knows the definitions from class but not how to prove the given theorem. After receiving grades on written work, the students are then given the chance to take the feedback, restructure their work to incorporate the new ideas, and re-submit a polished write-up for partial credit. Editing an existing solution allows the student a chance to focus on her writing as communication to a particular reader. The editing cycle has the happy side-effect of diverting the students' energy away from whining about grades and into processing feedback and correcting errors. Leading up to each test, in small groups, the students produce self-contained write-ups to some of our largest theorems, which we call Group Solutions. This assignment involves synthesizing several lemmas and thinking carefully about what the reader really needs to know to understand a proof. In general, students are willing to model the teaching behaviors described above, but we reinforce these behaviors with our grading scheme. In particular, we allot a substantial portion of the grade for the quality of the student's contributions in class and always ask for summaries of the key theorems' proofs on tests.

This course changes the way that the students think about mathematics; even more, our former students encourage everyone they know to take our course so that we can change how they think about mathematics as well.

Making students synthesize, structure, and articulate ideas increases the depth of their understanding. This perspective, that the students' job is to teach, also has several additional benefits. (1) I am constantly asking the students to exhibit the behaviors I will test, so I always know how my students are progressing. Moreover, the students like all the opportunities to practice the behaviors that I will test. (2) This perspective also encourages the student to make sure an answer is reasonable and teaches him the skills for learning on his own (rather than just learning what is told to him). All of these are the real goals of a mathematics class and much more useful to someone than having memorized some trigonometry identities. (3) And finally, students love it; they see that they learn much more by doing, so they always want to have the chalk in their hands. My fellow graduate students even tell me that it affects my students' behavior in office hours in future semesters.

I love teaching; helping students learn is a fascinating puzzle and one of my greatest passions. I learn by doing, and I intend to keep teaching for the rest of my life.