MATH 392: HOMOTOPY TYPE THEORY, PROBLEM SET #1

ANDREW J. BLUMBERG

1. PROBLEMS

(1) Express initial and terminal objects as colimits and limits, respectively.
(2) How unique is a limit? Explain.
(3) Work out explicit descriptions in the category of sets for the limits and colimits associated to the diagrams

$$X \underset{\longrightarrow}{\longrightarrow} Y$$

and

$$
\begin{array}{c}
X \\
\downarrow \\
Y \longrightarrow Z.
\end{array}
$$

(4) Show that the tensor product is the coproduct in the category of commutative rings.
(5) A monomorphism in a category $\mathcal{C}$ is an arrow $f \colon X \to Y$ such that given arrows $g, h \colon A \to X$, if $f \circ g = f \circ h$ then $g = h$. Compare the notion of monomorphism to the notion of injective defined in the context of Lawvere's framework for set theory.
(6) Prove that the set with one element has a single element and that the set arising in the subset classifier axiom has two elements, in Lawvere's framework.
(7) Construct $\mathbb{Q}$ in terms of Lawvere's setup.
(8) Show that the disjoint union of two sets exists in Lawvere's framework.

E-mail address: `blumberg@math.utexas.edu`