MATH 392: HOMOTOPY TYPE THEORY, PROBLEM SET #2

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1. Problems

(1) Describe an algorithm for converting any propositional formula into an equivalent formula in CNF. Prove the correctness of the algorithm. How much larger can the resulting formula be in the worst case?
(2) Prove that $p_1 \vee p_2 \vee \ldots \vee p_n \models q$ if and only if $(p_1 \vee p_2 \vee \ldots \vee p_n \vee \neg q)$ is unsatisfiable.
(3) (a) Prove that the resolution rule is sound.
    (b) Describe an algorithm for using the resolution rule to determine if an expression $(p_1 \vee p_2 \vee \ldots \vee p_n \vee \neq q)$ is unsatisfiable. (Hint: you must use proof by contradiction, i.e., try to derive $x \vee \neg x$.)
    (c) Prove that your algorithm will always terminate.
    (d) Prove that your algorithm is complete and sound.
    (e) (**Hard**) What can you say about the running time of this algorithm?
(4) Using Coq or Agda, express a formal proof of any tautology in propositional logic.

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Date: September 9, 2014.