MATH 392: HOMOTOPY TYPE THEORY, PROBLEM SET #5

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1. Problems

(1) Write out the formal description of a group object categorically (in terms of data in a cartesian-closed category) and as a simply-typed λ-calculus. Work out carefully what happens when we take the internal language of the categorical formulation or the associated category of the λ-calculus.
(2) Let $\mathcal{C}$ be a cartesian-closed category. A natural numbers object in $\mathcal{C}$ is determined by an initial object in the category of diagrams $* \to X \to X$.
   (a) Show that $\mathbb{N}$ is a natural numbers object in the category of sets.
   (b) Prove that if $\mathcal{C}$ has a natural numbers object, so does $\mathcal{C}/X$ for any object $X \in \mathrm{ob}(\mathcal{C})$.
   (c) Prove that if $\mathcal{C}$ has a natural numbers object, so does the "polynomial category" $\mathcal{C}[x]$ for an indeterminate arrow $x\colon A \to B$.
   (d) What is the corresponding structure in a simply-typed λ-calculus?
   (e) Prove an elaboration of the equivalence between cartesian-closed categories and simply-typed λ-calculi involving natural numbers objects.
(3) Read chapter 7 of May's "Concise course".

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