A CHARACTERIZATION OF THE PROJECTION LATTICE OF A VON-NEUMANN ALGEBRA

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Here is a dedication

Abstract. Let $\mathcal{L}$ be a non-empty family of projections on a separable Hilbert space. It is proved that $\mathcal{L}$ is the projection lattice of a von-Neumann algebra provided $\mathcal{L}$ is closed in the strong operator topology, and R belongs to $\mathcal{L}$ if R is a spectral projection of either a real linear combination of elements of $\mathcal{L}$, or of $i(PQ - QP)$, where P and Q are in $\mathcal{L}$ with $PQ + QP = P + Q - \frac{P \vee Q}{2}$.

1. Introduction

Throughout, let H denote a separable complex Hilbert space; $B(H)$ denotes the $*$-algebra of bounded linear operators on H. Recall that $\mathcal{N} \subset B(H)$ is called a von-Neumann algebra if $\mathcal{N}$ is a $*$-closed unital subalgebra of $B(H)$, which is closed in the Strong Operator Topology (the SOT). An operator P in $B(H)$ is called a *projection* if P is self-adjoint and $P^2 = P$. Of course then P is the orthogonal projection onto its range.

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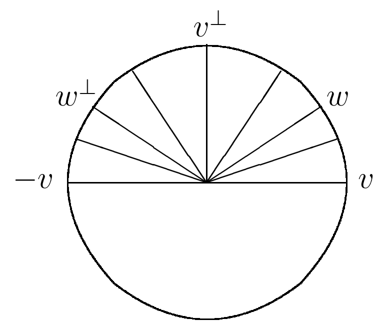
Proof of Lemma **??**. We thus obtain 16 points on $\mathbb{T}$, by bisecting the lines between v and w, v and w^+, etc.



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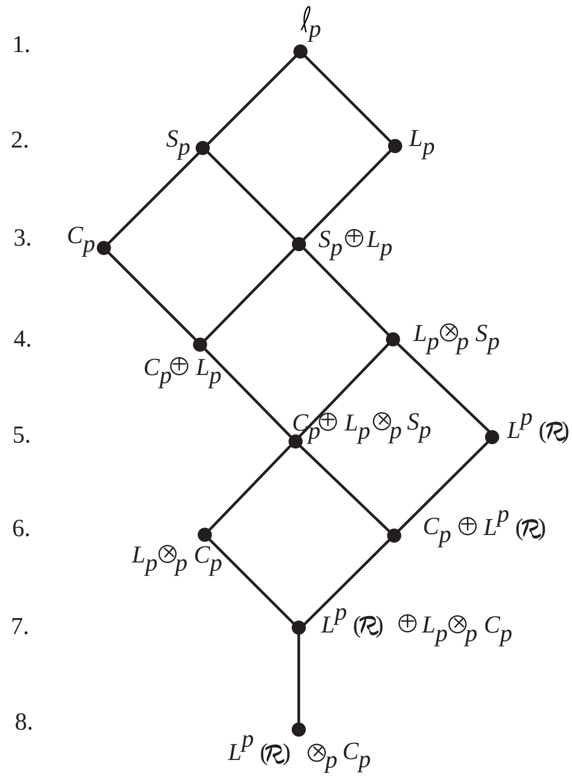

FIGURE 1

Continuing to take bisections, we obtain the claim. Rigorously, let e_1, e_2 be the Gram-Schmidt orthonormalization of v and w; i.e.,

$\square$

We now use Lemma **??** to prove the following

Claim. *For all real θ,*

$$e_1 + e^{i\theta} e_2 \in X \ . \tag{1}$$

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