# M316K – Foundations of Arithmetic

Spring 2009

Exam 1 – Version P

You have **50 minutes** to take this exam. No books, notes, calculators, or other electronic devices are allowed. Please write everything you want me to grade in your blue book; you will be allowed to take these questions with you when you are done. On the front of your blue book, please indicate which five questions you want graded. If you don't choose which questions I should grade, then I will choose for you at random. Also, please sign the upper right corner of your blue book; by your signature, you affirm the following Honor Pledge:

*"I pledge that I will neither give nor receive any unauthorized help on this exam. I will not use any books, notes, calculators, or other electronic devices while taking this exam. I will not attempt to look at any other student's paper, nor will I engage in behavior that will put me at risk of accidentally seeing another student's paper."*

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**PART A: Reading.** Please answer **one** of the following questions. Keep in mind that your goal is to demonstrate that you have read the material and understood the important points, so don't spend time trying to craft an exquisitely written essay. There is no length limit for this question, but you should easily be able to fit your response on one page.

**A1.** What are the NCTM process standards? (That is, what defines something as being a "process standard"?) Name three of the NCTM process standards, and briefly describe (in a few words) what each of them entails. Given that teachers are under increasing pressure to prepare students for multiple-choice standardized tests, which process standard do you think is most difficult to reinforce, and why?

**A2.** Give (but do not solve) two significantly different examples of problems, suitable for young students, that would encourage them to engage in algebraic thinking but do not require formal knowledge of algebra to solve. For each problem, briefly describe the kind of algebraic thinking that is involved, and explain how the problem is mathematically beneficial for the student to work on.

**PART B: Explorations.** Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

**B1.** In this problem, please give all answers in New Alphabitian (NAS). You may perform calculations in Hindu-Arabic numerals if you wish.

**(a)** How many days are in a week? (Assume that Alphabitian days and weeks are the same as ours.) Explain how you got your answer.
**(b)** What is the number after BBB? Explain how you got your answer.
**(c)** What is the number after ADD? Explain how you got your answer.
**(d)** What is the number before D0D0? Explain how you got your answer.
**(e)** Charlotte has CB acres of land, and David has DC acres of land. If Eric seizes both Charlotte's and David's land, how many acres of land will he gain in all? Show your work.

**B2.** The following are sets of input and output values from the game "What's My Rule?". For each set of input and output values, determine what the rule is, explain the rule both in English and as a formula, and determine the missing values. Show your work.

**(a)**

| Input | 2 | 4 | 7 | 8 | 11 | 15 | ? |
|---|---|---|---|---|---|---|---|
| Output | 1 | 7 | 16 | 19 | 28 | ? | 100 |

**(b)**

| Input | 1 | 2/3 | 1/5 | 6/7 | 7/11 | 5/12 | ? |
|---|---|---|---|---|---|---|---|
| Output | 3/2 | 9/4 | 15/2 | 7/4 | 33/14 | ? | 1 |

**B3.** Solve the following $3 \times 3$ magic square; that is, find all of the missing numbers. Explain your steps as you go. You may use everything you know about $3 \times 3$ magic squares. (*Hint:* Let $x$ be the number in the lower left corner, and let $y$ be the number in the lower right corner. Find the value of $y$ first.)

| | | |
|---|---|---|
| **42** | | |
| **37** | | |
| | **51** | |

**PART C: Problem Sets.** Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

**C1.** The University of Texas was established in 1883. Write this number using Roman numerals, and then using either Mayan or Babylonian numerals (your choice, but indicate which you are using).

**C2.** At Isaac Newton High School, students have the opportunity to take a wide selection of advanced math and science courses, including calculus, physics, and statistics. Answer the following questions:

**(a)** Consider the set of students who take calculus and physics, but not statistics. Represent this set using symbols, and then using a Venn diagram. (If you use letters to abbreviate the sets, be sure to explain what the letters mean!)
**(b)** Consider the set of students who take exactly one of these three courses. Represent this set using symbols, and then using a Venn diagram.

**C3.** Cheapco Cellular offers a cell phone plan in which users can make calls anywhere, anytime, for 2.7 cents per minute. A new competitor in the telecommunications industry, Eulertel, offers a plan in which users can make calls anywhere, anytime, for 2.2 cents per minute; however, users must pay a $2-per-month service charge for this plan.

**(a)** Represent the monthly cost of each cell phone plan as an equation. (You'll need one equation for Cheapco, and one for Eulertel.) Be sure to say what the variables in your equations represent.
**(b)** Represent the monthly cost of each cell phone plan as a graph. (Again, you'll have two graphs: one for Cheapco, and one for Eulertel.)
**(c)** Your mother e-mails you and tells you that she is unhappy with her cellular carrier, and wants to switch to either Cheapco or Eulertel. Under what circumstances would you advise her to switch to Cheapco? Under what circumstances would you advise her to switch to Eulertel? Be as precise as possible.