

M316K – Foundations of Arithmetic
Spring 2009
Exam 1 – Version X

You have **50 minutes** to take this exam. No books, notes, calculators, or other electronic devices are allowed. Please write everything you want me to grade in your blue book; you will be allowed to take these questions with you when you are done. On the front of your blue book, please indicate which five questions you want graded. If you don't choose which questions I should grade, then I will choose for you at random. Also, please sign the upper right corner of your blue book; by your signature, you affirm the following Honor Pledge:

"I pledge that I will neither give nor receive any unauthorized help on this exam. I will not use any books, notes, calculators, or other electronic devices while taking this exam. I will not attempt to look at any other student's paper, nor will I engage in behavior that will put me at risk of accidentally seeing another student's paper."



PART A: Reading. Please answer **one** of the following questions. Keep in mind that your goal is to demonstrate that you have read the material and understood the important points, so don't spend time trying to craft an exquisitely written essay. There is no length limit for this question, but you should easily be able to fit your response on one page.

- A1.** Name and briefly describe (in a sentence or two each) three numeration systems introduced in Section 2.3 other than the Hindu-Arabic system. Which of these systems is the most sophisticated, and why? (Give two reasons.)
- A2.** One thing that makes mathematics different from most other disciplines is the heavy use of notation (symbols that appear in mathematics but not in other disciplines). For many students, learning mathematical notation is one of the most challenging parts of learning the subject. What purposes does notation serve in mathematics? (Give two.) Give two examples of situations or problems in which mathematical notation plays a useful role.

PART B: Explorations. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

- B1.** Suppose that a certain number of people are gathered in a room. These people greet each other by shaking each other's hands; everybody shakes hands with everybody else exactly once (as in Exploration 1.1(2)).
- (a) Suppose there are five people in the room. How many handshakes take place in all? Solve this problem in **two** different ways; explain what you are doing each time.
- (b) Suppose there are 20 people in the room. How many handshakes take place in all? Show your work.
- (c) Suppose there are 20 people in the room, but instead of shaking hands, the people greet each other by punching each other in the face. (These are violent people.) Everybody punches everybody else in the face exactly once. How many face-punches take place in all? If you answered both this question and the previous question correctly, your answers should be different; explain why.
- B2.** The following are sets of input and output values from the game "What's My Rule?". For each set of input and output values, determine what the rule is, give the rule **as a formula**, and determine the missing values. **Show your work.**

(a)

Input	0	1	2	3	4	5	6	?
Output	0	4	10	18	28	40	?	130

(b)

Input	0	1	2	3	4	5	12	?
Output	-12	-7	-2	3	8	13	?	1988

B3. Answer the following questions about numbers in bases other than 10. **For each question, please explain how you got your answer and/or show your work.**

- (a) Consider the base-20 numeral $1f5_{20}$. Write this number in expanded form, and then write the number in base 10.
- (b) What is the number immediately after 36066_7 ? Give your answer in base 7.
- (c) What is the number immediately before $b0b0_{15}$? Give your answer in base 15.
- (d) Add the numbers 434_8 and 516_8 , and give your answer in base 8.

PART C: Problem Sets. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

C1. Your friendly instructor was born in the year 1980. Write this number in **two** of the following: Roman numerals, Mayan numerals, and Babylonian numerals. When writing the number in Mayan or Babylonian numerals, show your work.

C2. Use Venn diagrams to determine whether each argument below is valid or invalid. You don't have to give any explanation, but you may do so if you want me to have a better idea of your thought process in case your answer is wrong.

- (a) "All insects have six legs. All six-legged animals are invertebrates. Therefore, all insects are invertebrates."
- (b) "Some millionaires are hard-working. Some millionaires are criminals. Therefore, all hard-working people are criminals."
- (c) "All UT students are Longhorns fans. No UT students are Sooners fans. Therefore, no Longhorns fans are Sooners fans."

C3. Solve the following addition puzzles:

- (a) Find a three-digit number and a two-digit number such that the digits of the two numbers are 1, 3, 6, 7, and 8, in some order, and the sum of the two numbers is exactly 700. Explain how you solved the problem; if your solution involved guessing, try to find a way to do the problem without guessing. (There are several possible answers; you only need to give one.)
- (b) Find two **different** three-digit numbers which are "reverses" of each other (that is, they contain the same digits but in the opposite order) and add up to exactly 1776. Explain how you solved the problem; if your solution involved guessing, try to find a way to do the problem without guessing.