M316K – Foundations of Arithmetic

Spring 2009

Exam 1 – Version Y

You have **50 minutes** to take this exam. No books, notes, calculators, or other electronic devices are allowed. Please write everything you want me to grade in your blue book; you will be allowed to take these questions with you when you are done. On the front of your blue book, please indicate which five questions you want graded. If you don't choose which questions I should grade, then I will choose for you at random. Also, please sign the upper right corner of your blue book; by your signature, you affirm the following Honor Pledge:

"I pledge that I will neither give nor receive any unauthorized help on this exam. I will not use any books, notes, calculators, or other electronic devices while taking this exam. I will not attempt to look at any other student's paper, nor will I engage in behavior that will put me at risk of accidentally seeing another student's paper."

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PART A: Reading. Please answer **one** of the following questions. Keep in mind that your goal is to demonstrate that you have read the material and understood the important points, so don't spend time trying to craft an exquisitely written essay. There is no length limit for this question, but you should easily be able to fit your response on one page.

A1. The notion of "place value" is key to students' understanding of numbers and how we perform operations on those numbers. Explain in a sentence or two what the phrase "place value" means. Why is it important for children to understand the concept of place value? Give an example of a mistake that a child might make that could be caused by a lack of understanding of place value.

A2. One of the NCTM process standards involves coming up with different representations of a mathematical idea or problem. Give an example of an idea or problem that has at least three different representations, and describe three representations of that idea or problem. Of these representations, which do you think is the most suitable for young students? Explain.

PART B: Explorations. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

B1. In this problem, please give all answers in New Alphabitian (NAS). You may perform calculations in Hindu-Arabic numerals if you wish.

(a) How many days are in the month of December? (Assume that Alphabitian days and months are the same as ours.) Explain how you got your answer.
(b) What is the number immediately after DDD? Explain how you got your answer.
(c) What is the number immediately before BB00? Explain how you got your answer.
(d) Compute the sum of ACC and ADD. Show your work.

B2. Larry, Marlene, and Nita go to McBurger, a nearby fast food joint, for lunch. Larry orders a burger and an order of fries and pays \$3.10. Marlene orders a burger and a milkshake and pays \$3.90. Nita, a vegetarian, pays \$2.80 for an order of fries and a milkshake. Answer the following questions **without guessing or setting up equations with variables**. Explain your solution to each part.

(a) Which costs more: an order of fries or a milkshake? What is the difference in cost between the two?
(b) If Cody joins them and orders a burger, an order of fries, *and* a milkshake, how much will he pay? (*Hint:* What happens if you add Larry's, Marlene's, and Nita's orders together?)
(c) Find the cost of each individual item: a burger, an order of fries, and a milkshake.

B3. In the following problems, Ignacio, an expert dart thrower, throws a certain number of darts at a dartboard. You should assume throughout that each of Ignacio's darts hits the dartboard, and that he is allowed to hit a region multiple times. The number in a region indicates the number of points Ignacio receives for hitting that region. **In each of the following problems, show your work. If you are able to determine the answer without working out all the possibilities, explain why your answer is correct.**

(a) Suppose that Ignacio has two darts, and the dartboard is divided into regions marked 3, 5, and 7. What are the possible total scores that Ignacio can get?
(b) Suppose the dartboard is the same as in (a), but this time Ignacio has three darts. What are the possible total scores that he can get?
(c) Suppose that Ignacio has four darts, and the dartboard is divided into regions marked 1, 4, 7, and 10. What are the possible total scores that Ignacio can get?

PART C: Problem Sets. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

C1. At a certain elementary school, some students enjoy playing Xbox, some students enjoy playing with their iPhones, and some students enjoy playing T-ball. (Some students enjoy more than one of these, and some students don't enjoy any of these.)

(a) Consider the set of students who enjoy playing Xbox or enjoy T-ball, but do *not* enjoy playing with iPhones. Represent this set of students using symbols, and then using a Venn diagram. If you use letters to represent the sets, be sure to say what set each letter stands for.
(b) At this school, any student who does not play T-ball is considered "cool" (T-ball is *soooo* last century). If a student does play T-ball, he/she also has to enjoy playing Xbox *and* playing with his/her iPhone in order to be considered "cool." Represent the set of "cool" students using symbols, and then using a Venn diagram.

C2. Perform the following additions in bases other than ten. Show your work. On each problem, give your answer in the base in which the given numbers are written. (For example, if the two numbers are in base 8, give your answer in base 8.)

(a) $235_8 + 334_8$
(b) $100101_2 + 1011_2$
(c) $37a_{16} + 4ba_{16}$

C3. Answer the following questions. On each question, show your work.

(a) How many rectangles are in the 1×6 rectangle below on the left? (Remember, a square counts as a rectangle.)
(b) How many rectangles are in the 2×6 rectangle below on the right?
(c) How is your answer to (b) related to your answer to (a)? Explain why this relationship occurs.