

M316K – Foundations of Arithmetic
Spring 2009
Exam 2 – Version X

You have **50 minutes** to take this exam. No books, notes, calculators, or other electronic devices are allowed. Please write everything you want me to grade in your blue book; you will be allowed to take these questions with you when you are done. On the front of your blue book, please indicate which five questions you want graded. If you don't choose which questions I should grade, then I will choose for you at random. Also, please sign the upper right corner of your blue book; by your signature, you affirm the following Honor Pledge:

“I pledge that I will neither give nor receive any unauthorized help on this exam. I will not use any books, notes, calculators, or other electronic devices while taking this exam. I will not attempt to look at any other student's paper, nor will I engage in behavior that will put me at risk of accidentally seeing another student's paper. I will stop working immediately when time is called.”



PART A: Reading. Please answer **one** of the following questions. Keep in mind that your goal is to demonstrate that you have read the material and understood the important points, so don't spend time trying to craft an exquisitely written essay. There is no length limit for this question, but you should easily be able to fit your response on one page.

- A1.** Describe two algorithms for performing arithmetic operations that might be easier to learn than the standard algorithms that are normally taught in school, and explain why they might be easier. Give the names of the algorithms if you can remember them, but even if you remember the names, you must briefly describe the algorithms. Your algorithms should be for two **different** operations, and at least one of these operations must be multiplication or division.
- A2.** Suppose you are working with a student who has just learned what negative numbers are, but does not yet know how to add or subtract them. Suppose you want to teach this child how to do the addition problem $6 + (-3)$ and the subtraction problem $(-5) - (-7)$. For each of these problems, give an explanation using manipulatives or pictures (your choice) of how to do the problem. Remember, this student understands basic models for addition and subtraction, but doesn't know (and won't trust) the algebraic rules you're familiar with, such as “subtracting is the same as adding the opposite.”

PART B: Explorations. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

- B1.** Consider the multiplication problem 23×31 . Show how to perform this operation using the standard algorithm, and then model the operation using base-ten blocks. (Don't worry about drawing the blocks in full detail; just make sure your picture shows what you're thinking.) Show how the steps in the standard algorithm correspond to things that we see or do when we use the manipulatives. (For full credit, show at least two correspondences.)
- B2.** If we divide 85 by 12, we get a quotient of 7 and a remainder of 1. Answer the following questions:
- (a) Write a story problem that can be solved by dividing 85 by 12 so that the correct answer is 7. No further explanation is required, but make sure your problem is clear.
 - (b) Write a story problem that can be solved by dividing 85 by 12 so that the correct answer is 8. No further explanation is required, but make sure your problem is clear.
 - (c) Determine whether your two problems are “partitioning” problems or “repeated subtraction” problems. No explanation is required. (It's okay if your two problems are of the same type.)

B3. In Exploration 4.3, you discovered a powerful connection between the prime factorization of a number, and the number of divisors (or factors) of that number. Answer the following questions:

- (a) What is the prime factorization of 144? Show your work.
- (b) How many divisors does the number 144 have? Explain your answer.
- (c) How many of these divisors are even? Explain your answer.
- (d) Give an example of an integer that has exactly 20 divisors. You don't have to multiply the number out; you can leave it in prime factorization form. If you're confident that your answer is correct, you don't need to give any explanation, but if you don't, I can't give any partial credit if you are wrong. (This question is unrelated to the previous three.)

PART C: Problem Sets. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

C1. Perform the following subtractions in bases other than ten. Show your work. On each problem, give your answer in the base in which the given numbers are written. (For example, if the two numbers are in base 8, give your answer in base 8.)

- (a) $671_8 - 245_8$
- (b) $528_{13} - 3aa_{13}$
- (c) $20100_3 - 11211_3$

C2. In this problem, the letters A , B , C , D , E , and F all denote different digits. If all of the following arithmetic facts are true, determine what digit each letter represents. Explain your problem-solving process; you don't have to write a lot of explanation, but I need to be able to tell from your work what you are doing.

$$\begin{aligned} A - B = B & & A + C = DB & & B \times E = DF \\ BF \div A = E & & C \times E = AF & & E + F = E \end{aligned}$$

C3. Answer the following questions about GCFs (greatest common factors) and LCMs (least common multiples):

- (a) Find the GCF and LCM of 36 and 60 (and be sure to indicate which is which). You may use any method you wish, but make sure it's clear what you are doing.
- (b) Find two integers, other than 30 and 360, such that the GCF of these numbers is 30 and the LCM of these numbers is 360. Show your work. If you find the two integers by guessing, explain what knowledge you used to narrow down the possibilities you had to consider. (*Hint:* You may find it useful to represent the prime factorizations of the two unknown numbers using a Venn diagram.)