M316K – Foundations of Arithmetic

Spring 2009

Exam 2 – Version Y

You have **50 minutes** to take this exam. No books, notes, calculators, or other electronic devices are allowed. Please write everything you want me to grade in your blue book; you will be allowed to take these questions with you when you are done. On the front of your blue book, please indicate which five questions you want graded. If you don't choose which questions I should grade, then I will choose for you at random. Also, please sign the upper right corner of your blue book; by your signature, you affirm the following Honor Pledge:

"I pledge that I will neither give nor receive any unauthorized help on this exam. I will not use any books, notes, calculators, or other electronic devices while taking this exam. I will not attempt to look at any other student's paper, nor will I engage in behavior that will put me at risk of accidentally seeing another student's paper. I will stop working immediately when time is called."

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PART A: Reading. Please answer **one** of the following questions. Keep in mind that your goal is to demonstrate that you have read the material and understood the important points, so don't spend time trying to craft an exquisitely written essay. There is no length limit for this question, but you should easily be able to fit your response on one page.

A1. Define each of the following terms or phrases as precisely as you can: *a divides b*, *prime number*, *unique prime factorization* (be sure to explain what the word "unique" means here), *least common multiple*. On the definition of "a divides b," please say something more than "a is a factor of b" or "b is divisible by a"; explain what these ideas mean.

A2. What is the difference between a *model* for an operation and an *algorithm* for an operation? Give an example of a model for multiplication (don't just give me the name; show me what it is) and an algorithm for multiplication (again, I want more than the name).

PART B: Explorations. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

B1. Show three different ways, other than the standard subtraction algorithm, to perform the subtraction $711 - 458$. You don't have to write any explanation on these, as long you write enough work for me to see what you are doing. Which of these methods do you believe is easiest, and why?

B2. Answer the following questions. On each question, show your work.

(a) Gina and Harry are obsessive e-mail checkers. Gina checks her e-mail regularly every 40 minutes, and Harry checks his e-mail regularly every 36 minutes. If Gina and Harry both check their e-mail at exactly 9:34 AM, when will be the next time that they simultaneously check their e-mail? (Tell me what time it will be, not just how long it will take.)

(b) Suppose Kirsten wants to tile a 54-inch-by-78-inch rectangle with square tiles. All the square tiles are to be the same size, with side length a whole number of inches. Kirsten isn't allowed to overlap or break the tiles, or leave tiles hanging off the edges of the rectangle. What is the largest possible side length the tiles can have? How many of these tiles would she need? (See the diagram on the board.)

B3. Show how to solve each of the following problems using the "chip model" for addition and subtraction that we worked with in Explorations 5.1 and 5.2. Be sure to explain briefly what is going on in each step; pictures by themselves are not enough. Remember, your explanation must show how the answer naturally arises from the model, without using algebraic rules such as "subtracting a negative is the same as adding a positive."

(a) $-7+3$
(b) $-2-(-5)$

PART C: Problem Sets. Please answer **two** of the following questions. Do not mix-and-match parts of different questions; if you choose to answer a question, you are expected to answer all parts of that question.

C1. Consider the division problem $414 \div 3$. Show how to perform this operation using the standard algorithm, and then model the operation using base-ten blocks. (Don't worry about drawing the blocks in full detail; just make sure your picture shows what you're thinking.) Show how the steps in the standard algorithm correspond to things that we see or do when we use the manipulatives. (For full credit, show at least two correspondences.)

C2. A base-7 multiplication table (with a couple of entries missing) is given below:

$\times$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	11	13	15
3	0	3	6	12	15	x	24
4	0	4	11	15	22	26	33
5	0	5	13	21	26	y	42
6	0	6	15	24	33	42	51

(a) Give the two missing entries in the table (marked by x and y). Show your work; or if you are able to find an answer without doing any work, briefly explain how you got it. If you wish, you may use any of the information already given in the table.
(b) Using an algorithm of your choice, find the product $64_7 \times 3_7$. Show your work.
(c) Using an algorithm of your choice, find the product $215_7 \times 42_7$. Show your work.

C3. In each of the following problems, find digits having the indicated properties. In some cases, there may be more than one possible answer, but you only have to find one. Show your work and/or reasoning.

(a) Find a digit A such that the number $78A1$ is divisible by 9.
(b) Find digits B and C such that the number $8B3C$ is divisible by 36.
(c) Find a digit D such that the number $4092D$ is divisible by 7.